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RE TR 71-39

EVALUATION OF EXPERIMENTAL DRIVE SPRINGS FOR THE XM19 RIFLE

U.S. ARMY WEAPONS COMMAND

TECHNICAL REPORT

HENRY P. SWIESKOWSKI
AND
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August 1971

RESEARCH DIRECTORATE

WEAPONS LABORATORY AT ROCK ISLAND

RESEARCH, DEVELOPMENT AND ENGINEERING DIRECTORATE

U. S. ARMY WEAPONS COMMAND

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13. ABSTRACT

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3. Dynamic Loading						
4. Wave Propagation						
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Laboratory tests and a theoretical study were conducted to determine the optimum design for increasing the life of the XM19 drive spring.

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It was determined from this investigation that of the eight experimental designs that were evaluated, the two-piece spring assembly was superior because it retained maximum loads at the completion of the endurance tests.

CONTENTS

SUBJECT

Experimental drive springs for the XM19 rifle.

OBJECTIVE

The objectives of this evaluation were to obtain endurance data on XM19 drive springs under simulated firing conditions, to conduct a comparative study on the endurance properties of the various spring designs, to make recommendations for future development, and to conduct an analytical study including derivation of mathematical formulae for calculation of the dynamic behavior of helical compression springs and to apply the findings of the study for recommendation of an improved XM19 drive spring.

INTRODUCTION

The drive spring in the XM19 rifle functions in a dual capacity, as a drive spring and as a firing pin spring. The reasons for the spring operational failures are that appreciable spring set and load loss during firing result in insufficient spring energy for primer ignition. The spring energy required for reliable firing pin operation is 85 - 120 inch-ounces.

DISCUSSION

1. Spring Endurance Tests

The experimental XM19 drive springs were endurance-tested on the Spring Fatigue Tester. This tester was designed to simulate the actual firing conditions in the weapon. The velocities, accelerations, and deflections imparted to the drive spring in the weapon are reproduced on the tester. The duplication of the kinematic conditions is produced by a rotating cam with specially designed follower and guiding components. The design of the cam is based on a typical time-displacement curve of the spring motion in the weapon. A photograph of the tester showing the spring specimen and guiding components is included as Figure 1 in Appendix A.

Eight different XM19 drive spring designs were endurance-tested. Each spring was cycled 10,000 times. Free height and load readings were taken before testing and at the end of every 1000 compressions. The minimum test height was initially 5.60 inches and was increased during the tests to allow for a greater spring solid height. Two of the drive spring configurations were designed and developed by the Research Directorate, Weapons Laboratory at Rock Island. One of these was fabricated from a seven-wire strand of music wire

material and was the first design to be endurance-tested. The other USAWECOM drive spring design was based on a 14-wire strand of stainless steel material and was the final design to be evaluated. Drawings and complete specifications of each of the eight designs tested are given in Appendix A. A brief description of the pertinent characteristics of each experimental design is as follows:

DESIGN 1

Outside coil diameter = .440 in. ± .007

Number of wires in strand = 7

Strand diameter = .080 in.

Free height = 14.78 in.

Load = deflection rate = 2.0 lb/in.

Material - music wire

Number of springs tested = 8 springs

DESIGN 2

This design was a two-piece spring assembly in series, the short spring was installed at the moving end of the assembly:

	Long Spring	Short Spring
Outside coil diameter	.457 in. .007	.430 in. .005
Number of wires in strand	3	1
Strand diameter	.075	.059
Free height	15.63 in. .25	3.00
Load deflection rate	2.4 lb/in.	-
Material	music wire	music wire
Number of springs tested	2	2

DESIGN 3

Outside coil diameter = .451 in. ± .005

Number of wires in strand = 3

Strand diameter = .073 in.

Free height = 17.31 in.

Load-deflection rate = 1.4 lb/in.

Material = music wire, oil-tempered

Number of springs tested = 3

DESIGN 4

Outside coil diameter = .500 in.

Number of wires in strand = 7

Strand diameter = .097 in.

Free height = 13.25

Load-deflection rate = 3.3 lb/in.

Material = music wire, oil-tempered

Number of springs tested = 3

DESIGN 5

Outside coil diameter = .451 in. ± .005

Number of wires in strand = 3

Strand diameter = .073 in.

Free height = 17.31 in.

Load-deflection rate = 1.4 lb/in.

Material = silicone chrome, oil-tempered

Number of springs tested = 3

DESIGN 6

Outside coil diameter = .5105 in. ± .0025

Number of wires in strand = 3

Strand diameter = .102 in.

Free height = 13.405

Load-deflection rate = 3.1 lb/in.

Material = titanium

Number of springs tested = 3

DESIGN 7

This design was identical to Design 3 except that the first 20 coils at the moving spring-end were coated in a plastic material.

DESIGN 8

Outside coil diameter = .440 in. .007

Number of wires in strand = 14

Strand diameter = .100

Free height = 18.53

Load-deflection rate = 1.1

Material = stainless steel, type 302

Number of springs tested = 2

NOTE: This design represents a new type of strand construction for U. S. weapon spring applications. Heretofore, U. S. stranded wire springs were either a 3-wire or a 7-wire configuration. This spring and Design 1 were designed by the Research Directorate of the Weapons Laboratory at Rock Island.

Detailed test results of the springs are given in the tables in Appendix A. Measured values for free height and loads at two test-heights are given prior to testing and at the end of every 1000 cycles. The total reductions in free height and loads after the 10,000 cycles are also listed. Some springs attained their original free height after 10,000 compressions, but a loss in load values occurred. This loss can

be attributed to the fact that the individual wires in the strand became loose during the test with a resultant decrease in the load deflection rate. With respect to the two test-loads measured, the load at the maximum test height of 9.600 inches is the more important one since it indicates the amount of spring energy for firing pin activation. A low load-value at this height may result in failure to fire because of insufficient spring energy.

The three springs fabricated of titanium material (Design 6) are of interest since only a small loss in free height and load values occurred. Titanium is regarded as an unsuitable spring material, and little, if any, literature is available on springs formed of titanium. Titanium has rather low physical properties and is not recommended for highly stressed applications.

2. Theoretical Study

The theoretical study of the dynamic response of springs was performed at the University of Illinois under Contract DAAF03-69-C-0092. Time-displacement photographs were taken of impact-loaded springs and test results were correlated with theoretical findings. This study is included as Appendix B of this report.

CONCLUSIONS

No broken springs occurred; all springs satisfactorily sustained the 10,000 cycle test.

The design that retained the maximum load readings at 9.600 inches throughout the 10,000-cycle test was Design 2, the two-piece spring in series. The load values were approximately 50 per cent higher for this design than for any other design. The two-piece spring also incurred the greatest load loss during the test. However, the original load measurements of this spring were sufficiently large to ensure that final measurements were the maximum of all the experimental designs. The high loads that Design 2 exhibited are required for elimination of misfires because of insufficient spring energy.

This is the first time that a stranded spring of more than 7 wires has been used in a U. S. weapon. Previously, stranded wire springs for U. S. weapons have been of a 3-wire or a 7-wire construction. With respect to the two 14-wire springs evaluated in this test, one spring withstood the test exceptionally well.

Several graphs were prepared in the theoretical study to illustrate the relationship of the important variables. In the study, a plot of

the shearing stress in the wire with respect to the ratio Δ/h, where Δ represents the spring deflection and h is the free height of the spring is shown as Figure 6. This relationship is shown with the ratio c/r acting as a parameter; here c is the radius of the wire diameter and r is the radius of the coil diameter. The stress decreases as the ratio c/r decreases for the same value of Δ/h (Figure 6). Hence, to reduce operating stress levels, the use of multiple springs with smaller wire sizes is necessary. The 14-wire strand spring design that was endurance-tested on the Spring Fatigue Tester represents the maximum number of wires that can be satisfactorily fabricated for the space-load conditions of the XM19 drive spring application.

Theoretical and test results obtained from the study of the dynamic response of springs were correlated and were in close agreement. Differences between the theoretical and the test values were less than 1 per cent.

RECOMMENDATIONS

Based on the results obtained in this investigation, it is recommended that the two-piece design be considered for adoption as the standard drive spring for the XM19 Rifle.

APPENDICES

APPENDIX A

Figure 1. Photograpl Spring Fatigue Tester

Specification Sheet, Design 1, USAWECOM 7-Wire Strand Spring

Drawing B53099-10209, Design 2, 3-Wire Strand Spring Used in Series Combination

Drawing B53099-10278, Design 2, Single-Wire Spring Used in Series Combination

Drawing B53099-10096, Design 3, 3-Wire Strand Spring

Drawing B53099-10099, Design 4, 7-Wire Strand Spring

Drawing B53099-10114, Design 5, 3-Wire Strand Spring

Drawing B53099-10135, Design 6, 3-Wire Strand Spring

Specification Sheet, Design 8, USAWECOM, 14-Wire Strand Spring

TABLE	TITLE
I	Test Results, Spring Design 1
II	Test Results, Spring Design 2
III	Test Results, Spring Design 3
IV	Test Results, Spring Design 4
V	Test Results, Spring Design 5
VI	Test Results, Spring Design 6
VII	Test Results, Spring Design 7
VIII	Test Results, Spring Design 8

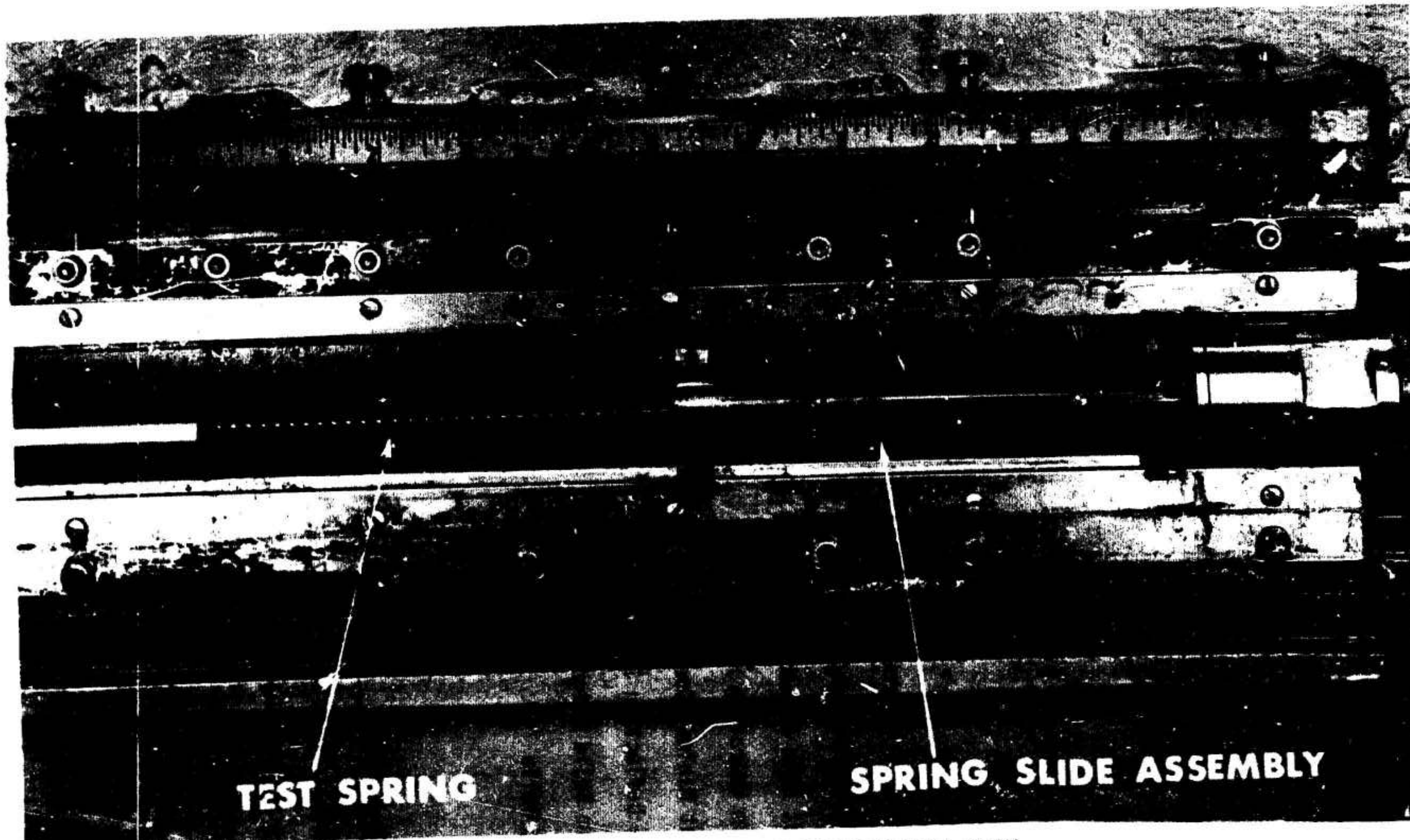


FIGURE 1

ENDURANCE TESTING OF EXPERIMENTAL SPIW
DRIVE SPRINGS ON THE SPRING FATIGUE TESTER

USAWECOM DESIGN

7-WIRE STRAND SPIW DRIVE SPRING

DESIGN 1

Wire Size	.029α .026*
Outside Diameter (in.)	.440 ± .007
Total Coils	63
Type of Ends	Closed and Silver-Soldered
Free Height, Approx. (in.)	14.78
Mean Assembled Height (in.)	9.600
Load at Mean Assembled Height (lb)	10.0 ± 1.0
Minimum Operating Height (in.)	5.600
Load at Minimum Operating Height (lb)	28.0 ± 4.0
Load - Deflection Rate (lb/in.)	2.0
Maximum Solid Height (in.)	5.50
Spring Helix	R. H.

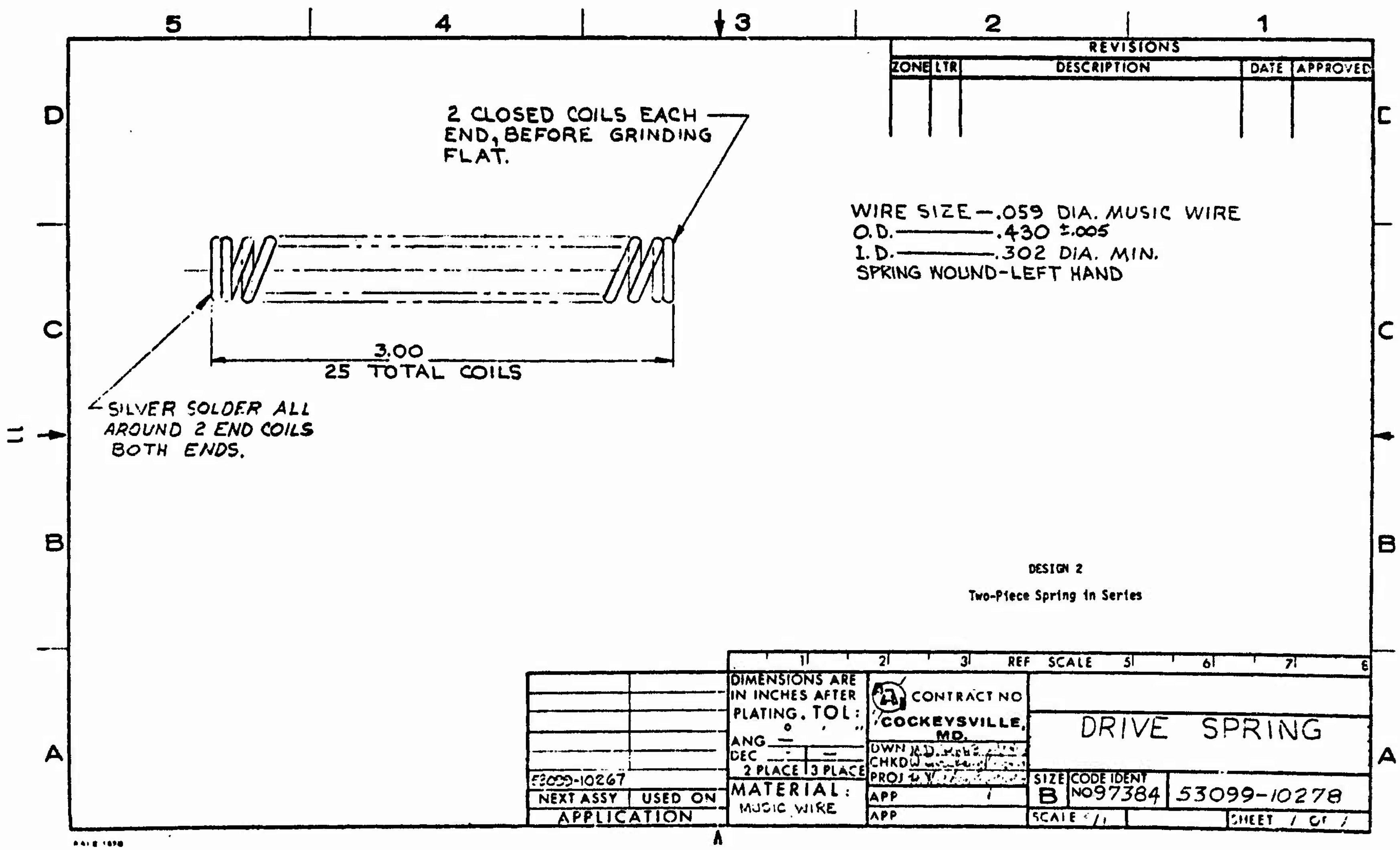
MATERIAL. Music Wire QQ 470

STRESS RELIEVE. Heat at 450° ± 25° for 30 minutes

PRESET. Compress to solid height 3 times prior to gaging

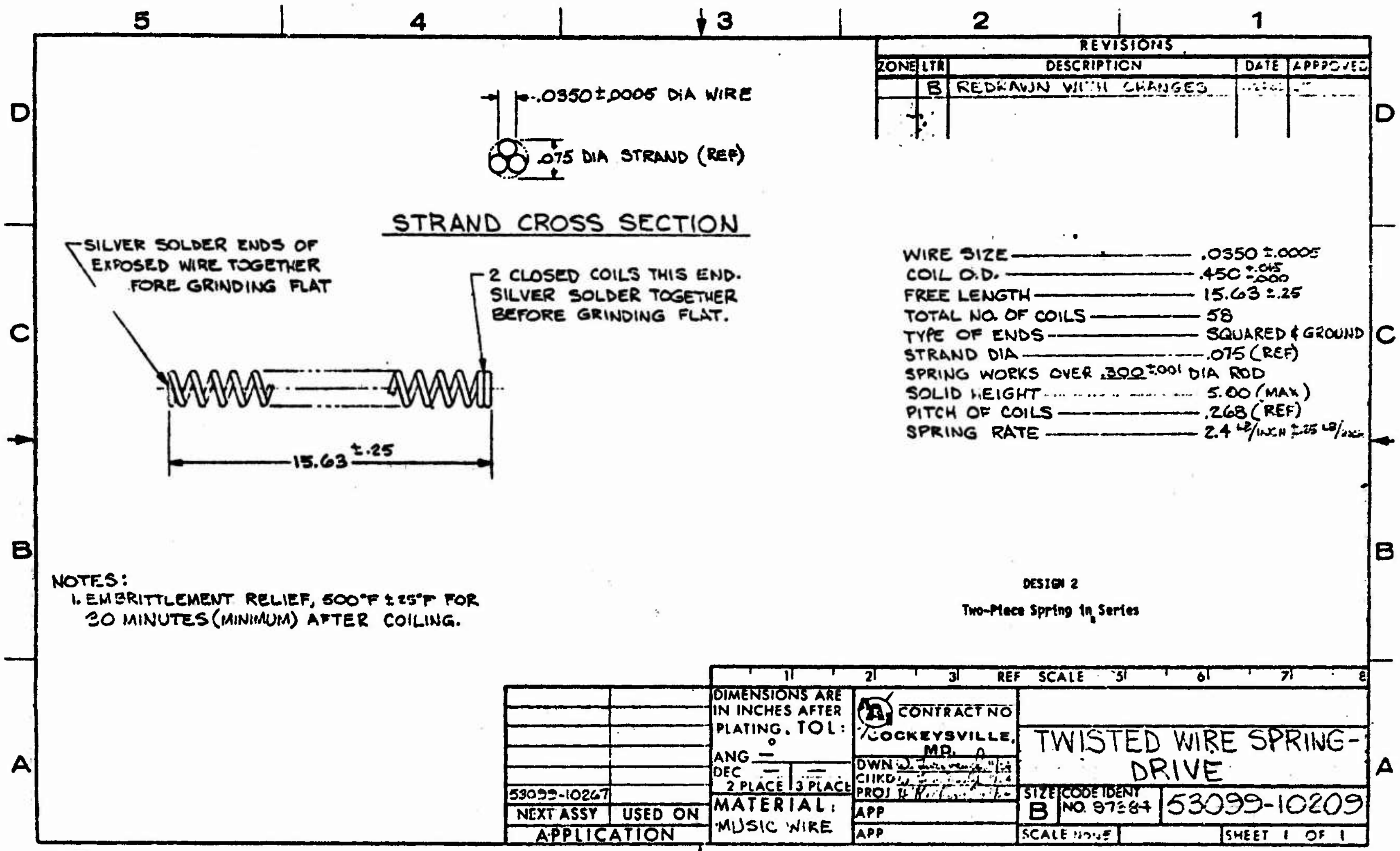
* .029 is diameter of core wire

* .026 is diameter of outer wire

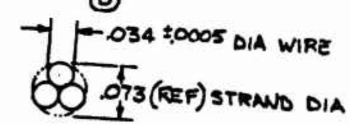


DESIGN 2

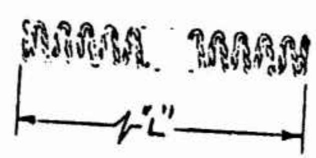
Two-Piece Spring in Series


.0350±.0005 DIA WIRE
.075 DIA STRAND (REF)
STRAND CROSS SECTION
SILVER SOLDER ENDS OF EXPOSED WIRE TOGETHER FORE GRINDING FLAT
2 CLOSED COILS THIS END. SILVER SOLDER TOGETHER BEFORE GRINDING FLAT.
15.63 ±.25
WIRE SIZE .0350 ±.0005
COIL O.D. .450 +.045 -.000
FREE LENGTH 15.63 ±.25
TOTAL NO. OF COILS 58
TYPE OF ENDS SQUARED & GROUND
STRAND DIA .075 (REF)
SPRING WORKS OVER .300 ±.001 DIA ROD
SOLID HEIGHT 5.00 (MAX)
PITCH OF COILS .268 (REF)
SPRING RATE 2.4 LB/INCH ±.25 LB/INCH
NOTES:
1. EMBRITTLEMENT RELIEF, 500°F ±25°F FOR 30 MINUTES (MINIMUM) AFTER COILING.
DESIGN 2
Two-Piece Spring in Series
REVISIONS
ZONE LTR DESCRIPTION DATE APPROVED
B REDRAWN WITH CHANGES
DIMENSIONS ARE IN INCHES AFTER PLATING. TOL:
ANG
DEC
2 PLACE 3 PLACE
MATERIAL: MUSIC WIRE
CONTRACT NO
COCKEYSVILLE, MD.
DWN
CHKD
PROJ
APP
APP
TWISTED WIRE SPRING-DRIVE
SIZE B
CODE IDENT NO. 97384
53099-10209
SCALE NONE
SHEET 1 OF 1
53099-10267
NEXT ASSY
USED ON
APPLICATION
REF SCALE

ZONE	LTR	REVISIONS DESCRIPTION	DATE	APPROVED
	A	ADDED NOTE 1 [illegible]	[illegible]	
	B	REVISED WIRE SIZE TOL. & [illegible] INFO.	[illegible]	[illegible]



STRAND CROSS SECTION



(A) WIRE SIZE - .034 ±.0005 DIA MUSIC WIRE (3 STRANDS) (B)
O.D. - - - - - - - .451 ±.005
LENGTH - "L" 17.31
TOTAL NO. COILS - 79
ENDS CLOSED - NOT GROUND
STRAND DIA. - .073 REF
WORKS OVER .300 DIA. ROD
SOLID HEIGHT 5.77
DEFLECTION 7.68 WITH 10.7 LBS. LOAD

NOTES:
1. EMBRITTLEMENT RELIEF, MIL-H-6875.

DESIGN 3

NEXT ASSY	USED ON
APPLICATION	

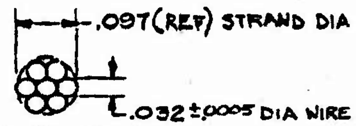
DIMENSIONS ARE IN INCHES AFTER PLATING. TOL:
ANG ± ~
DEC ±.06 (2 PLACE) ~ (3 PLACE)
MATERIAL: MUSIC WIRE, OIL TEMPERED

(A) COCKEYSVILLE, MD.
CONTRACT NO. [illegible]
DWN T. L. [illegible]
CHKD F. J. P. [illegible]
PROJ [illegible]
APP
APP

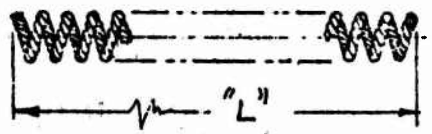
TWISTED WIRE SPRING DRIVE (3 STRAND)

SIZE B | CODE IDENT NO 97384 | 53099-10036
SCALE [illegible] | SHEET 1 OF 1

REVISIONS				
ZONE	LTR	DESCRIPTION	DATE	APPROVED
	A	13.25 W/ 5 13.54, 5.80 WAS 5.12, 50(48 ACTIVE COILS) WAS 59 &		
		DEFLECTION OF 2.95 WITH 9.7 LBS WAS DEFLECTION 3.42 WITH 9.7 LOAD		
		ADDED NOTE 3.	3/3/69	



STRAND CROSS SECTION



WIRE SIZE - .032±.0005 DIA. MUSIC WIRE (7 STRANDS)
COIL O.D. - - .500
(A) FREE LENGTH "L" - 13.25
(A) TOTAL NO. COILS - 50 (48 ACTIVE COILS)
ENDS CLOSED - NOT GROUND
STRAND DIA - - .097 (REF - 7 STRANDS)
TO WORK OVER .300 DIA. ROD
(A) SOLID HEIGHT - 5.80
(A) DEFLECTION 2.95 WITH 9.7 LBS LOAD

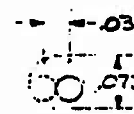
DESIGN 4

NOTES:
1. EMBRITTLEMENT RELIEF: MIL-H-6875
2. MATERIAL: MUSIC WIRE, OIL TEMPERED, QQ-W-470.
(A) 3. SOLDER OR BRAZE BOTH ENDS OF EXPOSED WIRE.

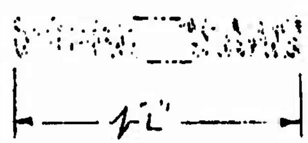
DIMENSIONS ARE IN INCHES AFTER PLATING. TOL: ANG ±½° DEC ±.06 (2 PLACE) ±.010 (3 PLACE)	CW 3096 CONTRACT NO. COCKEYSVILLE, MD.	TWISTED WIRE SPRING DRIVE (7 STRAND)	
	DWN T.L. SMITH 1/6/69 CHKD PROJ	SIZE B	CODE IDENT NO. 97384	53099-10099
MATERIAL: NOTE 2	APP	SCALE	SHEET 1 OF

NEXT ASSY	USED ON
APPLICATION	

ZONE	LTR	DESCRIPTION	DATE	APPROVED
REVISIONS				
		RELEASED	9/29/68	J.M.B
	A	REVISED NOTE 1	9-5-69	W.F.



STRAND CROSS SECTION



WIRE SIZE - .034 ±.0005 DIA
O.D. - - - - - - .451 ±.005
LENGTH - "L" - 17.31
TOTAL NO. COILS - 79
ENDS CLOSED - NOT GROUND
STRAND DIA. - .073 REF
WORKS OVER .300 DIA. ROD
SOLID. HEIGHT 5.77
DEFLECTION 7.68 WITH 10.7 LBS LOAD

NOTES

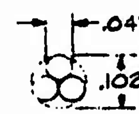
1. (A) STRESS RELIEVE AFTER COILING AT 310°F ±10°F FOR 1 HOUR.
2. SILICONE CHROME WIRE, OIL TEMP., SAE 9254
3. SOLDER OR BRAZE BOTH ENDS OF EXPOSED WIRE.

DESIGN 5

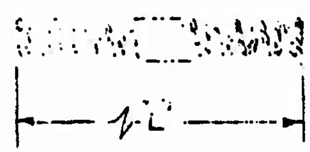
NEXT ASSY / USED ON — APPLICATION	DIMENSIONS ARE IN INCHES AFTER PLATING. TOL: ANG. ± DEC ± 2 PLACE / 3 PLACE — MATERIAL: NOTE 2	CW 3096. CONTRACT NO — COCKEYSVILLE, MD. — DWN / CHKD / PROJ / APP / APP
TWISTED WIRE SPRING-DRIVE (3 STRAND)	SIZE B — CODE IDENT NO 97384	53099-10114
SCALE		SHEET 1 OF 1

REVISIONS

ZONE	LTR	DESCRIPTION	DATE	APPROVED



STRAND CROSS SECTION



WIRE SIZE - .047 ±.0005 DIA TITANIUM WIRE (3 STRANDS)
O.D. - - - - - - .508 $^{+.005}_{-.000}$
LENGTH - "L" 13.405 ±.010
TOTAL NO. COILS - 56
ENDS CLOSED - NOT GROUND
STRAND DIA. - .102 REF
WORKS OVER .300 DIA. ROD
SOLID HEIGHT 5.71
TEST LOAD - 11.8 LBS AT 3.73 INCH DEFLECTION

DESIGN 6

NOTES:
1. MATERIAL: TITANIUM WIRE, (Ti-13V-11CR-3AL).
2. STRESS RELIEVE AFTER COILING.
3. SOLDER OR BRAZE BOTH ENDS OF EXPOSED WIRE.

NEXT ASSY	USED ON
APPLICATION	

DIMENSIONS ARE IN INCHES AFTER PLATING. TOL:
ANG. ±
DEC ±.04 (2 PLACE) | (3 PLACE)
MATERIAL: NOTE 1

CONTRACT NO
COCKEYSVILLE, MD.
DWN
CHKD
PROJ.
APP
APP

TWISTED WIRE SPRING - DRIVE (3 STRAND)

SIZE	CODE IDENT NO	
B	97384	53099-10135

SCALE | SHEET 1 OF 1

USAWECOM DESIGN

14-WIRE STRAND SPIW DRIVE SPRING

DESIGN 8

Wire Size (in.)	.020
Outside Diameter (in.)	.440 ± .007
Total Coils	60
Type of Ends	Closed and Silver-Soldered
Free Height, Approx. (in.)	18.53
Mean Assembled Height (in.)	9.60
Load at Mean Assembled Height (lb)	10.0 ± 1.0
Minimum Operating Height (in.)	5.60
Load at Minimum Operating Height (lb)	28.0 ± 4.0
Load - Deflection Rate (lb/in.)	1.12
Maximum Solid Height (in.)	5.55
Spring Helix	R. H.

MATERIAL. Stainless Steel, Type 302

STRESS RELIEVE. Heat at 900° ± 25° for 30 minutes

PRESET. Compress to solid height 3 times prior to gaging

TABLE I

USAMECOM DESIGN
7-WIRE STRAND, MUSIC WIRE MATERIAL
EXPERIMENTAL SPIN DRIVE SPRINGS

	NO. OF CYCLES	ORIGINAL	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	AMOUNT LOSS
	Free Height	14.56 in.	14.56	14.56	14.56	14.56	14.56	14.56	14.56	14.56	14.56	14.56	.00
SPRING 1	P @ 9.6"	12.0 lb.	12.0	10.8	10.3	10.2	10.2	10.2	10.2	10.2	10.2	10.0	2.0
	P @ 5.6"	31.0 lb.	31.0	28.8	26.0	25.6	24.8	24.8	24.6	24.6	24.6	24.6	6.4
	Free Height	14.50	14.50	14.50	14.50	14.50	14.50	14.50	14.50	14.50	14.50	14.50	.00
SPRING 2	P @ 9.6"	11.3	10.3	10.2	10.2	10.2	10.2	10.2	10.0	10.0	10.0	10.0	1.3
	P @ 5.6"	30.3	25.6	24.1	24.1	24.1	24.1	24.0	24.0	24.0	24.0	23.3	7.0
	Free Height	14.26	14.26	14.22	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20	.06
SPRING 3	P @ 9.6"	10.4	9.9	9.6	9.6	9.3	9.3	9.3	9.3	9.3	9.3	9.3	1.1
	P @ 5.6"	26.4	25.6	23.0	23.0	23.0	23.0	23.0	23.0	23.0	23.0	23.0	3.4
	Free Height	14.30 in.	14.30	14.30	14.30	14.30	14.30	14.30	14.30	14.30	14.30	14.30	.00
SPRING 4	P @ 9.6"	10.6 lb.	10.0	10.0	10.0	10.0	9.8	9.8	9.8	9.8	9.6	9.6	1.0
	P @ 5.6"	25.0 lb.	23.6	23.0	23.0	23.0	23.0	23.0	23.0	23.0	23.0	23.0	2.0
	Free Height	14.40	14.40	14.40	14.40	14.40	14.40	14.40	14.40	14.40	14.40	14.40	.00
SPRING 5	P @ 9.6"	10.8	10.3	10.0	9.8	9.8	9.6	9.6	9.6	9.6	9.6	9.6	1.2
	P @ 5.6"	26.0	26.0	26.0	24.0	24.0	23.8	23.8	23.8	23.6	23.6	23.6	2.4
	Free Height	14.46	14.46	14.46	14.46	14.46	14.46	14.46	14.46	14.46	14.46	14.46	.00
SPRING 6	P @ 9.6"	10.6	10.6	10.0	9.8	9.8	9.8	9.8	9.3	9.3	9.3	9.3	1.3
	P @ 5.6"	27.0	27.0	25.0	25.0	25.0	25.0	25.0	25.0	24.6	24.6	23.8	3.2
	Free Height	14.52 in.	14.52	14.52	14.52	14.52	14.52	14.52	14.52	14.52	14.52	14.52	.30
SPRING 7	P @ 9.6"	11.6 lb.	10.3	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	1.6
	P @ 5.6"	28.8 lb.	25.3	25.3	24.8	24.8	24.8	24.8	24.8	24.8	24.8	24.8	4.0
	Free Height	14.62	14.62	14.62	14.62	14.62	14.62	14.62	14.62	14.62	14.62	14.62	.00
SPRING 8	P @ 9.6"	11.6	11.0	10.6	10.6	10.6	10.6	10.0	10.0	1.0	10.0	10.0	1.6
	P @ 5.6"	27.3	26.6	26.6	26.0	25.6	25.0	24.0	24.0	24.0	24.0	24.0	3.3

TABLE II

SHORT SPRING, SINGLE WIRE, DWG. 53099-10278
LONG SPRING, 3-WIRE STRAND, DWG. 53099-10209
EXPERIMENTAL SPIW DRIVE SPRINGS

	NO. OF CYCLES	ORIGINAL	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	AMOUNT LOSS
SHORT SPRING 1 LONG SPRING 1	Free Height	3.00 in. 16.20 in.	3.00 14.90	3.00 14.84	3.00 14.80	3.00 14.80	3.00 14.80	3.00 14.80	3.00 14.72	3.00 14.70	3.00 14.70	3.00 14.70	.00 1.50
LOAD IN SERIES	P @ 9.6"	22.2 lb.	17.8	17.8	17.8	17.8	17.8	16.4	16.4	16.4	16.4	16.4	5.8
LOAD IN SERIES	P @ 6.25	34.4 lb.	32.7	32.7	32.7	32.7	32.7	32.4	32.4	32.4	32.4	32.4	2.0
SHORT SPRING 1 LONG SPRING 1	Free Height	3.00 16.34	2.93 15.20	2.93 15.16	2.93 15.08	2.93 14.94	2.93 14.86	2.93 14.86	2.93 14.80	2.93 14.80	2.93 14.78	2.87 14.78	.13 1.56
LOAD IN SERIES	P @ 9.6"	20.8	17.0	17.0	17.0	16.6	16.3	16.1	16.1	15.8	15.8	15.8	5.0
LOAD IN SERIES	P @ 6.25	33.7	30.7	30.7	29.3	28.7	28.7	28.5	28.5	28.5	28.5	28.0	5.7

TABLE III

3-WIRE STRAND, MUSIC WIRE MATERIAL, DWG. 53099-10096
EXPERIMENTAL SPIW DRIVE SPRINGS

	NO. OF CYCLES	ORIGINAL	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	AMOUNT LOSS
	Free Height	16.90 in.	16.90	16.80	17.70	16.68	16.68	16.66	16.66	16.64	16.64	16.62	.70
SPRING 1	P @ 9.6"	9.3 lb	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.6	8.6	8.6	.7
	P @ 6.25	17.6 lb	17.6	17.6	17.6 b	17.6	17.6	17.6	17.6	17.0	17.0	17.0	.6
	Free Height	17.80	17.40	17.38	17.34	17.34	17.30	17.30	17.20	17.20	17.20	17.16	.64
SPRING 2	P @ 9.6"	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	9.6	9.6	.4
	P @ 6.25	21.0	19.0	19.0	19.0	19.0	10.0	10.0	18.6	18.6	18.6	18.6	2.4
	Free Height	17.58	17.22	17.22	17.22	17.22	17.20	17.18	17.16	17.16	17.16	17.16	.42
SPRING 3	P @ 9.6"	11.4	11.1	11.1	11.1	11.1	10.5	10.5	10.5	10.5	10.5	10.5	.9
	P @ 6.25	18.2	18.0	17.3	15.5	15.5	15.5	15.5	15.5	15.5	15.5	15.5	2.7

TABLE IV

7-WIRE STRAND, MUSIC WIRE MATERIAL, DWG. 53099-10099
EXPERIMENTAL SPIW DRIVE SPRINGS

	NO. OF CYCLES	ORIGINAL	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	AMOUNT LOSS
	Free Height	13.00 in.	12.96	12.96	12.96	12.96	12.96	12.94	12.94	12.94	12.94	12.94	.06
SPRING 1	P @ 9.6"	10.8 lb.	10.6	10.6	10.6	10.6	10.6	10.6	10.6	10.6	10.6	10.6	.2
	P @ 6.25	29.0 lb.	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	.0
	Free Height	13.16	13.14	13.14	13.14	13.14	13.14	13.14	13.14	13.14	13.14	13.14	.02
SPRING 2	P @ 9.6"	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	.0
	P @ 6.25	32.3	31.8	31.8	31.8	31.8	31.3	31.0	31.0	31.0	31.0	31.0	1.3
	Free Height	13.20	12.70	12.70	12.70	12.70	12.70	12.70	12.70	12.70	12.70	12.70	.50
SPRING 3	P @ 9.6"	13.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	2.0
	P @ 6.25	27.0	24.6	24.6	24.6	24.6	24.6	24.6	24.6	24.6	24.6	24.6	2.4

TABLE V

3-WIRE STRAND, CHROME SILICONE MATERIAL, DWG. 53099-10114
EXPERIMENTAL SPIN DRIVE SPRINGS

	NO. OF CYCLES	ORIGINAL	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	AMOUNT LOSS
SPRING 1	Free Height	17.04 in.	17.00	17.00	17.00	16.94	16.94	16.92	16.92	16.92	16.92	16.92	.12
	P @ 9.6"	11.3 lb.	10.8	10.8	10.8	10.6	10.6	10.6	10.6	10.6	10.6	10.6	.2
	P @ 6.25	20.3 lb.	19.8	19.8	19.0	19.0	19.0	19.0	19.0	18.8	18.8	18.8	1.5
SPRING 2	Free Height	17.10	17.06	17.06	17.02	17.02	17.02	17.02	17.02	17.02	17.02	17.92	.08
	P @ 9.6"	11.6	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	11.0	.6
	P @ 6.25	19.8	19.8	19.8	19.7	19.8	19.8	19.8	19.8	19.8	19.8	19.8	.0
SPRING 3	Free Height	17.48	17.40	17.40	17.40	17.40	17.40	17.38	17.38	17.38	17.38	17.38	.10
	P @ 9.6"	13.3	12.0	12.0	12.0	11.8	11.8	11.8	11.8	11.8	11.5	11.5	1.8
	P @ 6.25	24.6	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	2.1

TABLE VI

3-WIRE STRAND, TITANIUM MATERIAL, DWG. 53099-10135
EXPERIMENTAL SPIN DRIVE SPRINGS

	NO. OF CYCLES	ORIGINAL	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	AMOUNT LOSS
	Free Height	13.50 in.	13.42	13.42	13.42	13.42	13.42	13.40	13.40	13.40	13.40	13.40	.10
SPRING 1	P @ 9.6"	8.6 lb.	8.6	8.6	8.6	8.6	8.6	8.6	8.3	8.3	8.3	8.3	.3
	P @ 6.25	18.8 lb.	18.8	18.8	18.8	18.8	18.8	18.8	18.8	18.8	18.8	18.8	.0
	Free Height	13.40	13.34	13.34	13.32	13.30	13.30	13.30	13.30	13.30	13.30	13.30	.10
SPRING 2	P @ 9.6"	8.6	8.3	8.3	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	.6
	P @ 6.25	19.0	18.8	18.8	18.8	18.8	18.6	18.6	18.6	18.6	18.6	18.6	.4
	Free Height	13.36	13.28	13.28	13.28	13.26	13.26	13.26	13.24	13.24	13.24	13.24	.12
SPRING 3	P @ 9.6"	8.0	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	.2
	P @ 6.25	18.6	18.6	18.6	18.6	18.3	18.3	18.3	17.8	17.8	17.8	17.8	.8

TABLE VII

PLASTIC-COATED SPRING
3-WIRE STRAND, MUSIC WIRE MATERIAL, DWG. 53099-10096

	NO. OF CYCLES	ORIGINAL	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	AMOUNT LOSS
	Free Height	17.62 in.	17.48	17.46	17.30	17.30	17.30	17.30	17.20	17.20	17.20	17.20	.42
SPRING 1	P @ 9.6"	11.5 lb.	11.0	11.0	11.0	11.0	11.0	11.0	10.5	10.5	10.5	10.5	1.0
	P @ 6.25	20.3 lb.	19.7	19.7	19.7	19.7	19.7	19.7	19.3	19.3	19.3	19.0	1.3

TABLE VIII

USAAVECOM DESIGN
14-WIRE STRAND, STAINLESS-STEEL MATERIAL
EXPERIMENTAL SPIW DRIVE SPRINGS

	NO. OF CYCLES	ORIGINAL	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	AMOUNT LOSS
	Free Height	19.14 in.	19.0	19.0	18.98	18.98	18.98	18.98	18.98	18.98	18.98	18.98	.16
SPRING 1	P @ 9.6"	12.8 lb.	11.0	10.4	10.0	10.0	9.7	9.5	9.5	9.5	9.5	9.5	3.3
	P @ 7.0	14.0 lb.	14.0	13.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	2.0
	Free Height	18.16	17.96	17.94	17.94	17.94	17.94	17.94	17.92	17.92	17.92	17.92	.24
SPRING 2	P @ 9.6"	12.0	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5	.5
	P @ 7.0	16.0	15.0	15.0	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	1.5

APPENDIX B

Report, "Dynamic Response of Springs," on the study performed at the University of Illinois under Contract DAAF03-69-C-0092

DYNAMIC RESPONSE OF SPRINGS

Introduction

In many problems of engineering significance, which involve the dynamics of springs, the mass of the spring can be neglected and it is usually assumed that the spring is linear. These assumptions allow one to formulate the problem so that in many cases a mathematical solution is possible. The problem, however, becomes much more complex if one considers the mass of the spring and assumes that large displacements are possible.

A. E. H. Love(1)* presents expressions for the static response of helical springs subjected to large deflections. In a more recent work, Curt I. Johnson(2) lists formulas and derivations of a design procedure for dynamically loaded extension and compression springs. In an article by J. Dick(3), the analysis of a simple shock wave in a helical spring is discussed. R. Geballe(4), reviews a theory presented by Krebs and Weidlich(5). This treatment is correct only when the spring extensions lie in a limited region near the unloaded length. N. J. Durant(6) derives an expression for the stress in a dynamically loaded spring in compression when the helix angle is small. In a paper by Y. Kagawa(7), the longitudinal and the torsional vibrations of helical springs of finite length with small pitch are analyzed. A wide-band, short-duration, pulse technique is used by W. Britton and G. Langley(8) to investigate the stress wave propagation in helical springs. In a paper by B. L. Johnson and E. E. Stewart(9) transfer functions are presented for helical springs.

Theory

As far as the static response of a spring is concerned, the following can be written:

$$\Delta = f_1 (F, T, E, \nu, h, c, r, p) \qquad (1)$$

where

Δ = deflection of the spring from its unstretched length (see Fig. 1)

F = axial force in the spring

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*Numbers in parentheses refer to List of References.

T = twisting moment applied to the spring

E = modulus of elasticity

ν = Poisson's ratio

h = unstretched length of the spring

c = radius of the circular cross section of the wire

r = radius of the spring in its unstretched position

and

p = pitch of the spring in its unstretched position.

Also,

$$\theta = f_2 (F, T, E, \nu, h, c, r, p) \tag{2}$$

where

θ = angle of twist of the spring.

It should be noted that the cross section of the wire is assumed to be circular. If this is not the case, then a characteristic length should be used instead of c. If, for example, a square cross sectional wire were being considered, the length of a side would be used.

Equation (1) can be written, after applying the theory of dimensional analysis, in the following form:

$$\frac{\Delta}{r} = f_3 \left(\frac{F}{Er^2}, \frac{T}{Er^3}, \nu, \frac{c}{r}, \frac{p}{r}, \frac{h}{r}\right) \tag{3}$$

However, since Δ is a linear function of h, Eq. (3) can be reduced to

$$\frac{\Delta}{h} = f_4 \left(\frac{F}{Er^2}, \frac{T}{Er^3}, \nu, \frac{c}{r}, \frac{p}{r}\right) \tag{4}$$

Similar remarks can be made concerning Eq. (2) and, hence,

$$\frac{\theta r}{h} = f_5 \left(\frac{F}{Er^2}, \frac{T}{Er^3}, \nu, \frac{c}{r}, \frac{p}{r}\right) \tag{5}$$

For a given spring, ν, $\frac{c}{r}$ and $\frac{p}{r}$ are constants and therefore, Eq. (4) and (5) can be written as:

$$\frac{\Delta}{h} = \epsilon = f_6 \left(\frac{F}{Er^2}, \frac{T}{Er^3}\right) \tag{6}$$

$$\frac{\theta r}{h} = \beta = f_7 \left(\frac{F}{Er^2}, \frac{T}{Er^3}\right) \tag{7}$$

where

$$\epsilon = \frac{\Delta}{h} = \text{strain in the spring}$$

and

$$\beta = \frac{\theta r}{h} = \text{rotational strain in the spring.}$$

If Eq. (6) and (7) are solved for T and F, the following results:

$$\frac{T}{Er^2} = f(\epsilon, \beta) \tag{8}$$

and

$$\frac{T}{Er^3} = g(\epsilon, \beta) \tag{9}$$

The equations which determine the dynamic response of a spring will now be derived. Figure 2 shows an element of the spring of original length dx at the original position x and then shows the same element of the spring at time t. A summation of forces in the x direction yields:

$$\frac{\partial F}{\partial \epsilon} \frac{\partial^2 u}{\partial x^2} + \frac{\partial F}{\partial \beta} \frac{\partial^2 \phi}{\partial x^2} = \frac{M}{h} \frac{\partial^2 u}{\partial t^2} \tag{10}$$

where

$u = u(x, t) =$ displacement of the spring

$\phi = \phi(x, t) =$ rotation of the spring about the x axis

$$\epsilon = \frac{\partial u}{\partial x} = \text{strain in the spring}$$

$$\beta = r\,\frac{\partial \phi}{\partial x} = \text{rotational strain in the spring}$$

and

$M =$ total mass of the spring.

A summation of moments about the x axis yields:

$$\frac{\partial T}{\partial \epsilon} \frac{\partial^2 u}{\partial x^2} + r\,\frac{\partial T}{\partial \beta} \frac{\partial^2 \phi}{\partial x^2} = \frac{Mr^2}{h} \frac{\partial^2 \phi}{\partial t^2} \tag{11}$$

Equations (8) and (9) determine the static response of springs. It will now be assumed that the static equations can be used to calculate the coefficients in the dynamic equations (10) and (11). If one considers a thin wire helical spring, Eqs. (8) and (9) can be determined from a theory given by Love[1]. The following equations can be written from the general theory of bending and twisting of thin rods in Love's work:

$$\frac{Fr^2}{EI} = \frac{r}{r_1} \frac{\cos \alpha_1}{(1+\nu)} \left[\frac{r}{r_1} \sin \alpha_1 \cos \alpha_1 - \sin \alpha \cos \alpha \right] - \frac{r}{r_1} \sin \alpha_1 \left[\frac{r}{r_1} \cos^2 \alpha_1 - \cos^2 \alpha \right] \tag{12}$$

and

$$\frac{Tr}{EI} = \frac{\sin \alpha_1}{(1+\nu)} \left[\frac{r}{r_1} \sin \alpha_1 \cos \alpha_1 - \sin \alpha \cos \alpha \right] + \cos \alpha_1 \left[\frac{r}{r_1} \cos^2 \alpha_1 - \cos^2 \alpha \right] \tag{13}$$

where

I = moment of inertia of the circular cross section

r = initial radius of the cylinder on which the helix lies

r_1 = final radius of the cylinder on which the helix lies

α = initial angle which the tangent makes with a plane perpendicular to the axis of the helix

and

α_1 = final angle which the tangent makes with a plane perpendicular to the axis of the helix.

If the initial pitch of the spring is p, then

$$p = 2\pi r \tan \alpha. \tag{14}$$

The final pitch p_1 is given by the expression

$$p_1 = 2\pi r_1 \tan \alpha_1. \tag{15}$$

Equations (12) and (13) determine the static response of a helical spring.

The geometric relation between the length of the wire L and the unstretched length of the spring h is

$$h = L \sin a \tag{16}$$

If the change in length of the wire is neglected, the following expression can be written

$$h_1 = L \sin a_1 \tag{17}$$

where h_1 is the final length of the spring and hence

$$\Delta = h_1 - h = L(\sin a_1 - \sin a) \tag{18}$$

The geometric relation between the length h, the radius r and the total helix angle γ is

$$\gamma = \frac{h}{r \tan a} \tag{19}$$

Also, the total final helix angle is

$$\gamma_1 = \frac{h_1}{r_1 \tan a_1} \tag{20}$$

Hence the angle of twist is given by the expression

$$\theta = \gamma_1 - \gamma = L \left[\frac{\cos a_1}{r_1} - \frac{\cos a}{r} \right] \tag{21}$$

Equation (18) and (21) can be written in the following dimensionless form:

$$\frac{\Delta}{h} = \frac{\sin a_1}{\sin a} - 1 \tag{22}$$

and

$$\frac{\theta r}{h} = \frac{1}{\sin a} \quad \frac{r}{r_1} \cos a_1 - \cos a \tag{23}$$

The curvature K and the torsion τ of a helix are given by the following equations

$$K = \frac{\cos^2 a}{r} \tag{24}$$

and

$$\tau = \frac{\sin a \ \cos a}{r} . \tag{25}$$

Also

$$K_1 = \frac{\cos^2 a_1}{r_1} \tag{26}$$

and

$$\tau_1 = \frac{\sin a_1 \ \cos a_1}{r_1} \tag{27}$$

where K_1 is the final curvature and τ_1 is the final torsion. The bending moment M and the torque $\overline{T}$ in the spring are related to the curvature K and the torsion τ by the equations

$$M = EI (K_1 - K) \tag{28}$$

and

$$\overline{T} = GJ (\tau_1 - \tau) \tag{29}$$

where J is the polar moment of inertia and G is the modulus of rigidity in shear. Hence, using strength of materials formulas for the bending and torsional stresses, σ and S, the following results:

$$\sigma = \frac{Mc}{I} = Ec \left[\frac{\cos^2 a_1}{r_1} - \frac{\cos^2 a}{r} \right] \tag{30}$$

and

$$S = \frac{\overline{T}c}{J} = Gc \left[\frac{\sin a_1 \ \cos a_1}{r_1} - \frac{\sin a \ \cos a}{r} \right] \tag{31}$$

Equations (30) and (31) can be written in the following dimensionless form:

$$\frac{\sigma}{E} = \frac{c}{r} \left[\frac{r}{r_1} \cos^2 a_1 - \cos^2 a \right] \tag{32}$$

and

$$\frac{S}{E} = \frac{c}{2r(1+\nu)} \ \frac{r}{r_1} \left[\sin a_1 \ \cos a_1 - \sin a \ \cos a \right] \tag{33}$$

Several plots of the preceding equations are shown in Fig. 3, 4, 5, 6, 7, 8, 9 and 10.

The quantities $\frac{\partial F}{\partial \epsilon}$, $\frac{\partial F}{\partial \beta}$, $\frac{\partial T}{\partial \epsilon}$ and $\frac{\partial T}{\partial \beta}$, which occur in the dynamic equations (10) and (11), will now be determined from the static equations. Since

$$\epsilon = \frac{\sin a_1}{\sin a} - 1 \tag{34}$$

and

$$\beta = \frac{1}{\sin a} \left[\frac{r}{r_1} \cos a_1 - \cos a \right] \tag{35}$$

equations (12) and (13) can be written as

$$\begin{aligned} \frac{Fr^2}{EI} = (\beta \sin a + \cos a) \sin a \Bigg[& - \nu \frac{(1+\epsilon)}{(1+\nu)} (\beta \sin a + \cos a) \\ & + \frac{(1+\epsilon)\cos^2 a}{\sqrt{1 - (1+\epsilon)^2 \sin^2 a}} - \frac{\cos a}{(1+\nu)} \Bigg] \end{aligned} \tag{36}$$

and

$$\begin{aligned} \frac{Tr}{EI} = \frac{(1+\epsilon)}{(1+\nu)} \sin^2 a \Big[(1+\epsilon) (\beta \sin a + \cos a) - \cos a \Big] & \\ + (\beta \sin a + \cos a) \left[1 - (1+\epsilon)^2 \sin^2 a \right] & \\ - \sqrt{1 - (1+\epsilon)^2 \sin^2 a} \, \cos^2 a & \end{aligned} \tag{37}$$

Hence Eq. (36) and (37) yield:

$$\begin{aligned} \frac{r^2}{EI} \frac{\partial F}{\partial \epsilon} = (\beta \sin a + \cos a) \sin a \Bigg[& \frac{-\nu}{(1+\nu)} (\beta \sin a + \cos a) \\ & + \frac{\cos^2 a}{\left[1 - (1+\epsilon)^2 \sin^2 a \right]^{3/2}} \Bigg] \end{aligned} \tag{38}$$

$$\begin{aligned} \frac{r^2}{EI} \frac{\partial F}{\partial \beta} = \frac{r}{EI} \frac{\partial T}{\partial \epsilon} = \sin^2 a \Bigg[& \frac{(1+\epsilon)\cos^2 a}{\sqrt{1 - (1+\epsilon)^2 \sin^2 a}} \\ & - \frac{\cos a}{(1+\nu)} - \frac{2(1+\epsilon)\nu}{(1+\nu)} (\beta \sin a + \cos a) \Bigg] \end{aligned} \tag{39}$$

and

$$\frac{r}{EI}\frac{\partial T}{\partial \beta} = \sin \alpha \left[1 - \frac{\nu (1+\epsilon)^2}{(1+\nu)} \sin^2 \alpha \right] \tag{40}$$

Letting

$$\frac{r}{h} = \bar{r} \tag{41}$$

$$\frac{x}{h} = \bar{x} \tag{42}$$

$$\frac{u}{h} = \bar{u} \tag{43}$$

$$\frac{t}{\sqrt{\frac{M h r^2}{EI}}} = \bar{t} \tag{44}$$

and

$$\frac{r}{h}\phi = \bar{v}, \tag{45}$$

Eq. (10) and (11) become

$$\frac{r^2}{EI}\frac{\partial F}{\partial \epsilon}\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{r^2}{EI}\frac{\partial F}{\partial \beta}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} = \frac{\partial^2 \bar{u}}{\partial \bar{t}^2} \tag{46}$$

and

$$\frac{r}{EI}\frac{\partial T}{\partial \epsilon}\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{r}{EI}\frac{\partial T}{\partial \beta}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} = \frac{\partial^2 \bar{v}}{\partial \bar{t}^2}. \tag{47}$$

Hence, the dynamic response of the spring reduces to the solution of Eq. (46) and (47) subject to the initial and boundary conditions. Once $\bar{u} = \bar{u}(x, t)$ and $\bar{v} = \bar{v}(x, t)$ are known, then the quantities $\epsilon = \frac{\partial u}{\partial x}$ and $\beta = r\frac{\partial \phi}{\phi x}$ can be computed and hence the stresses can be determined from Eq. (32) and (33). It should be noted that in the above theory it is assumed that the coils of the spring do not touch each other.

Equations (46) and (47) are nonlinear and coupled and hence it would be advantag eous if the equations could be simplified. It should be noted that in Eq. (38) and (39) the term β is multiplied by $\sin \alpha$. For most springs α is small and as can be seen from Fig. 4, the term β is small for the static case with zero T. Hence the term $\beta \sin \alpha$ will be neglected in the case of springs which are impacted by a mass. It also seems reasonable to assume that in the case the term $\frac{r}{EI}\frac{\partial F}{\partial \epsilon}$ is a constant and is given by the equation

$$\frac{r}{EI}\frac{\partial F}{\partial \epsilon} = \sin a \left[1 - \frac{\nu}{(1+\nu)} \cos^2 a \right] \tag{48}$$

Neglecting the term $\frac{r^2}{EI}\frac{\partial F}{\partial \beta}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}$, Eq. (46) becomes

$$\sin a \left[1 - \frac{\nu}{1+\nu} \cos^2 a \right] \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} = \frac{\partial^2 \bar{u}}{\partial \bar{t}^2} \tag{49}$$

which is the wave equation.

Hence, in the case of a helical spring, which is impacted by a mass, the problem reduces to the solution of Eq. (49) which satisfies the initial and boundary conditions. In order to check the validity of the above assumptions, tests were run on a helical spring which was initially compressed and impacted by a mass. The results were recorded by a streak camera. A schematic drawing of the apparatus is shown in Fig. 11.

The unstretched length of the spring was 18.25 in. and weighed 0.415 lb. The compressed length of the spring before impact was 17.70 in. Also, $r = 0.500$ in., $p = 0.315$ in. and $c = 0.050$ in. The spring was impacted by a mass which weighed 0.656 lb. at a velocity of about 300 in. per sec. The drum on the streak camera has a radius of 6 in. and was rotating at 2.94 revolutions per second. The results of the tests are shown in Fig. 12. Neglecting the rotation of the spring about the x axis, this figure gives the axial displacement of each element of the spring.

According to the theory, the equation

$$a^2 \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} = \frac{\partial^2 \bar{u}}{\partial \bar{t}^2} \tag{50}$$

where a is the wave speed and

$$a^2 = \sin a \left[1 - \frac{\nu}{1+\nu} \cos^2 a \right] \tag{51}$$

has to be solved with the following initial and boundary conditions:

$$\bar{u}(\bar{x}, 0) = \bar{x}\,\bar{\delta} \tag{52}$$

$$\dot{\bar{u}}(\bar{x}, 0) = 0 \tag{53}$$

$$\bar{u}(0, \bar{t}) = 0 \tag{54}$$

$$\bar{u}(1, \bar{t}) = g(\bar{t}) \tag{55}$$

Equation (52) expresses the condition that the spring has an initial displacement.

Equation (53) states that the spring has no initial velocity. Equation (54) implies no deflection at the end $\bar{x} = 0$ and Eq. (55) expresses the known deflection at the end $\bar{x} = 1$ as a function of time.

Letting

$$\bar{u}_1 = \bar{u} - \bar{x}\,\bar{\delta} \tag{56}$$

Eq. (50), (52), (53), (54) and (55) become

$$a^2 \frac{\partial^2 \bar{u}_1}{\partial \bar{x}^2} = \frac{\partial^2 \bar{u}_1}{\partial \bar{t}} \tag{57}$$

$$\bar{u}_1 (\bar{x}, 0) = 0 \tag{58}$$

$$\dot{\bar{u}}_1 (\bar{x}, 0) = 0 \tag{59}$$

$$\bar{u}_1 (0, t) = 0 \tag{60}$$

$$\bar{u}_1 (1, \bar{t}) = g(\bar{t}) - \bar{\delta} = \bar{h}(t) \tag{61}$$

The solution of Eq. (57) through (61) can be obtained, as shown in Fig. 12, by moving the displacement curve along the line with a velocity a. The calculated position of particular point on the spring is shown in Fig. 12.

For the given spring,

$$\tan \alpha = \frac{p}{2\pi r} = \frac{0.315}{2\pi \times 0.5} = 0.1002$$

Therefore $\sin \alpha = 0.0997$, $\cos \alpha = 0.995$ and $a = 0.283$. The time required for a disturbance to move from one end of the spring to the other is

$$\bar{t}_1 = \frac{1}{a} = 3.53$$

or

$$t_1 = \bar{t}_1 \sqrt{\frac{M h r^2}{EI}} = 0.02032 \text{ sec}$$

Hence, the velocity is

$$v = \frac{17.70 \text{ in.}}{0.02032} = 871 \text{ in/sec} \quad \text{(theory)}$$

From Fig. 12

$$t_1 = \frac{2.26}{12 \times \pi} \quad \frac{1}{2.94} = 0.02036 \text{ sec}$$

and hence the velocity is

$$v = \frac{17.70}{0.02036} = 870 \text{ in/sec (experiment)}$$

The quantity $\bar{\delta} = \frac{17.70 - 18.25}{18.25} = -0.0301$ and the time that the mass is in contact with the spring is (taken from Fig. 12)

$$T = \frac{9.40}{\pi\, 12} \quad \frac{1}{2.94} = 0.0848 \text{ sec}$$

and hence

$$\bar{T} = \frac{0.0848}{\sqrt{\frac{M h r^2}{EI}}} = 14.72$$

It should be noted, from Fig. 12, that some of the coils of the spring are in contact. This would tend to increase the stresses in the spring considerably since the area of contact is along a line. In order to examine this further, another picture was taken of the same spring in which the spring was compressed 5.50 in. before being impacted. The results are shown in Fig. 14 where the drum was rotating at 5.88 revolutions per sec. One can notice the large number of coils being compressed together thus increasing the stresses over a large length of the spring.

Summary and Conclusions

In the previous section, a theory was presented which would predict the dynamic response of springs. An experiment was performed in order to check the assumptions made in the theory. The experiment consisted of impacting a spring with a mass and recording the results with a streak camera. Figures 12 and 14 are two such recordings. In the theory, the assumption was made that the coils did not come in contact with each other. This assumption is valid if the spring is not impacted to too great an extent. As can be seen from Fig. 12 and 14, there was contact between the coils. It is felt that if the impact was not as large, their would be less discrepancy between theory and experiment. This contact between the coils occurs along a line and hence large stresses result. In Fig. 14, where the spring was initially compressed to a large degree, contact between the coils occurred over a large portion of the spring after impact. If these stresses are in the inelastic range, a permanent shortening of the spring would result after each impact. This means that even though the stresses are below the elastic limit in the static case when the coils are just touching, the stresses can be considerably above the elastic limit in the dynamic case.

In the static case, for a given stiffness one can reduce the stresses by making the spring out of several finer wires. A considerable reduction in the torsional stresses occurs and it is felt that this would also reduce the stresses in the dynamic case.

Another method for reducing the stresses would be to construct the spring so that the cross section of the spring wire would be rectangular. This would reduce considerably the stresses due to impacting of the coils on one another since the area of contact would be much larger.

The above theory would be modified so that contact between the coils could be taken into account in the dynamic case. Also expressions could be determined which would yield an expression for the stresses. It is apparent that any additional meaningful research in this area would have to take into account the contact between the coils. This could be a possible future area of investigation.

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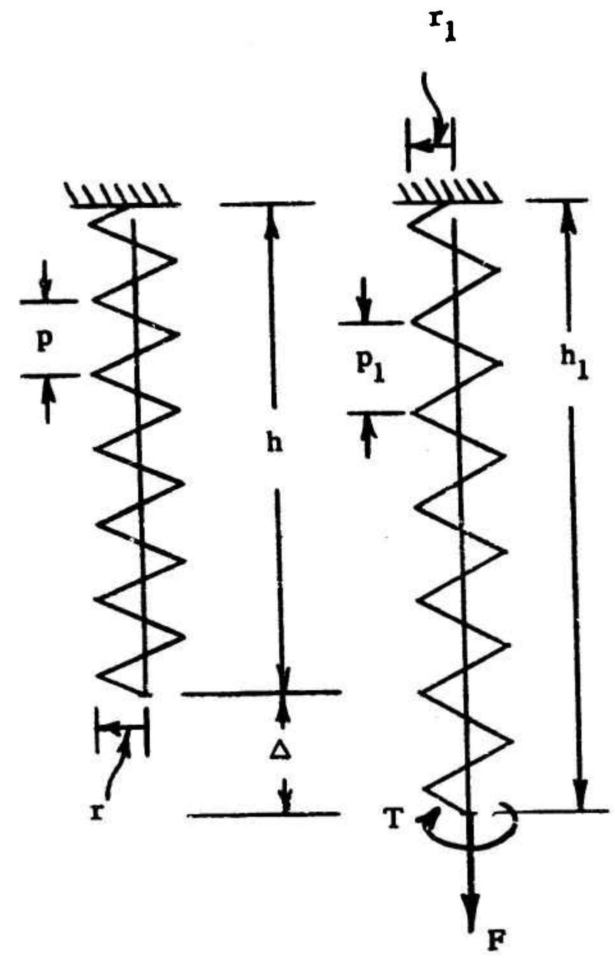


Fig. 1

Spring Configuration

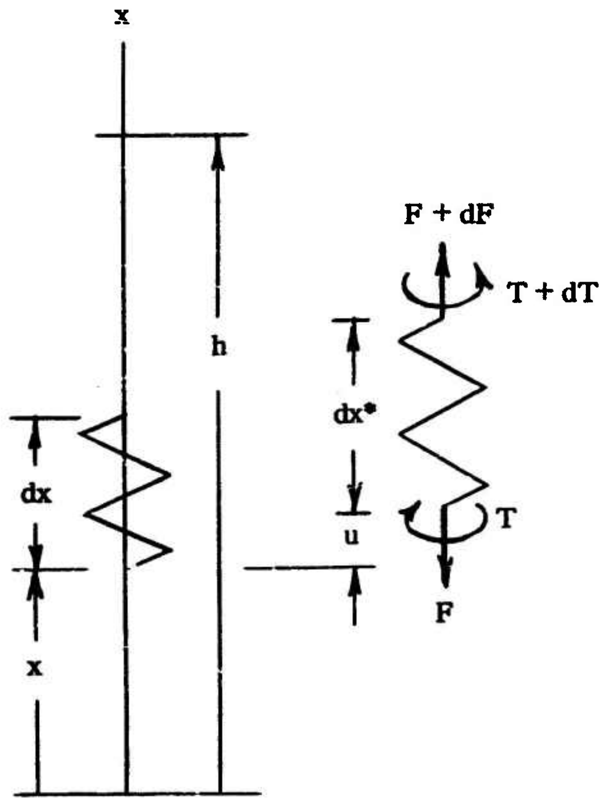


Fig. 2

Free Body Diagram for an Element of the Spring

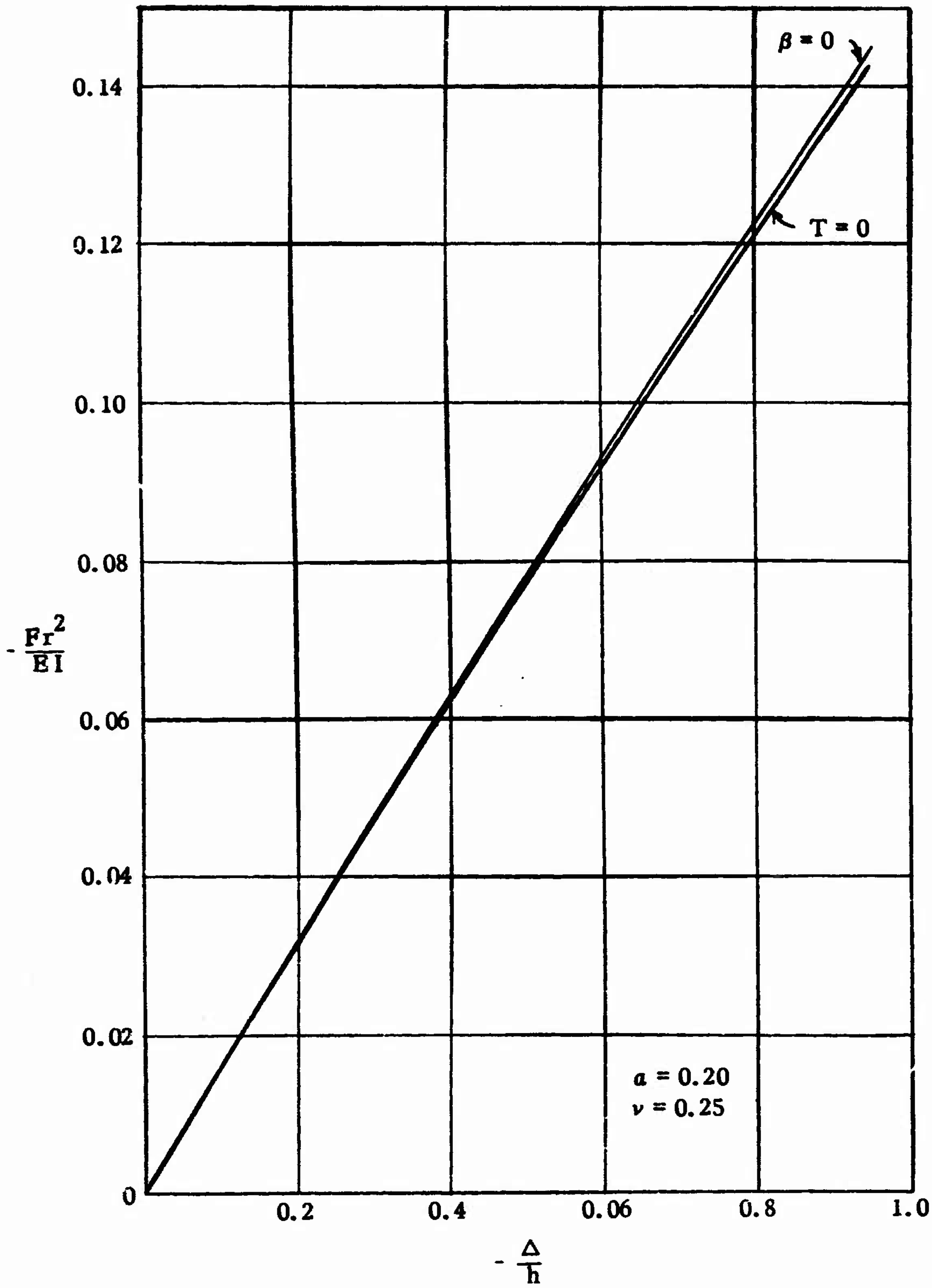


Fig. 3

Plot of $\frac{Fr^2}{EI}$ vs. $\frac{\Delta}{h}$

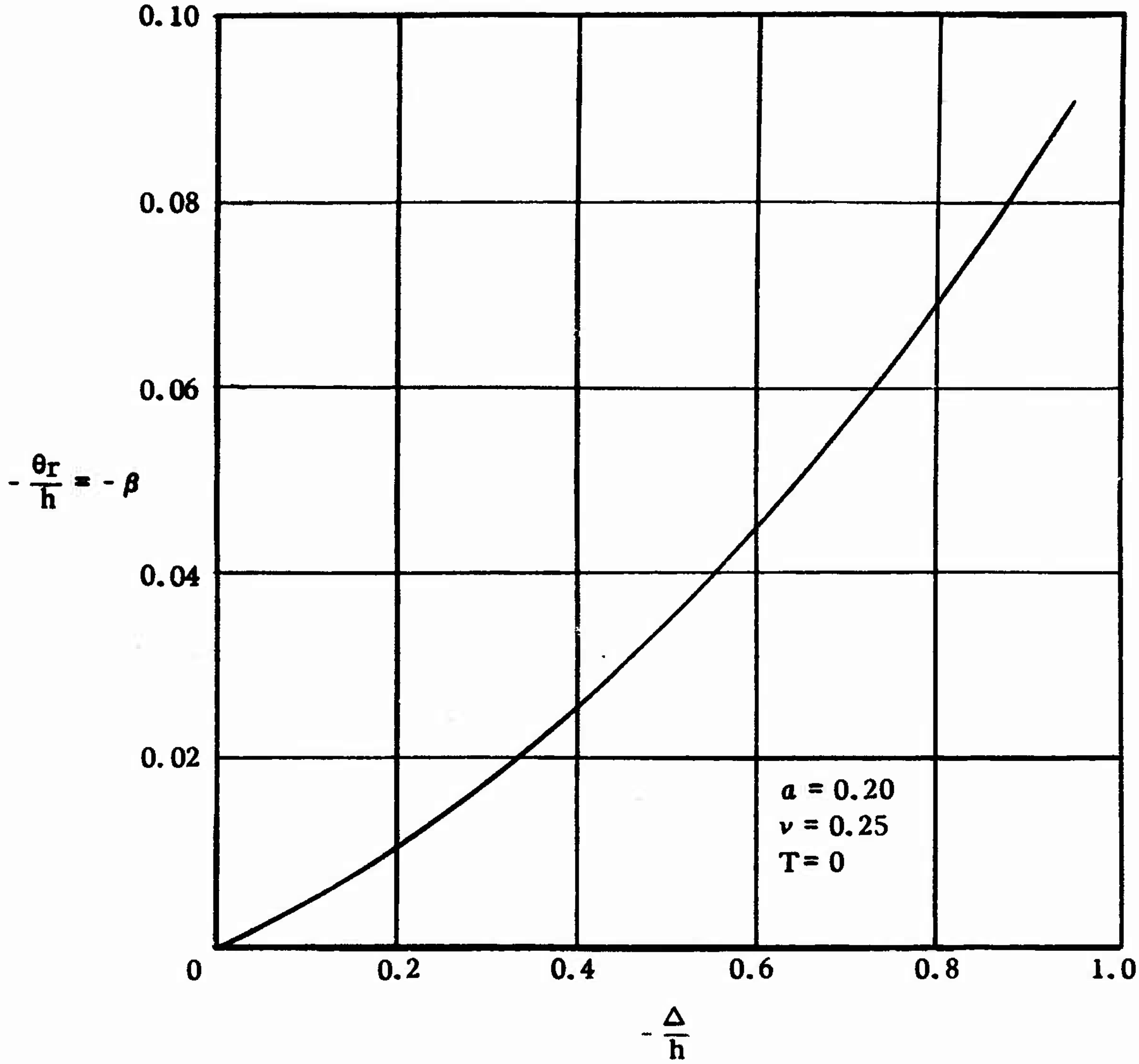


Fig. 4

Plot of β vs. $\frac{\Delta}{h}$

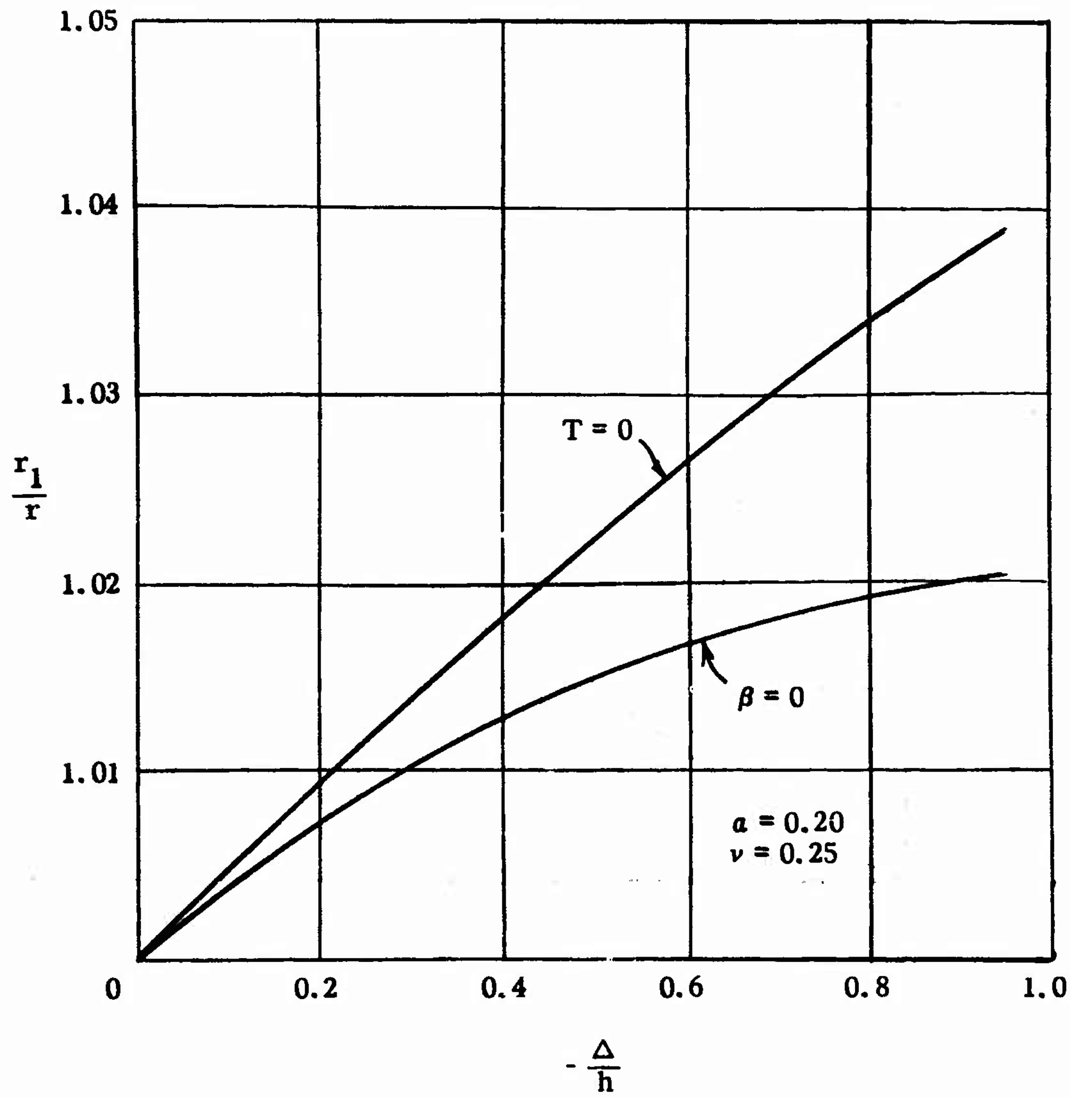


Fig. 5

Plot of $\frac{r_1}{r}$ vs. $\frac{\Delta}{h}$

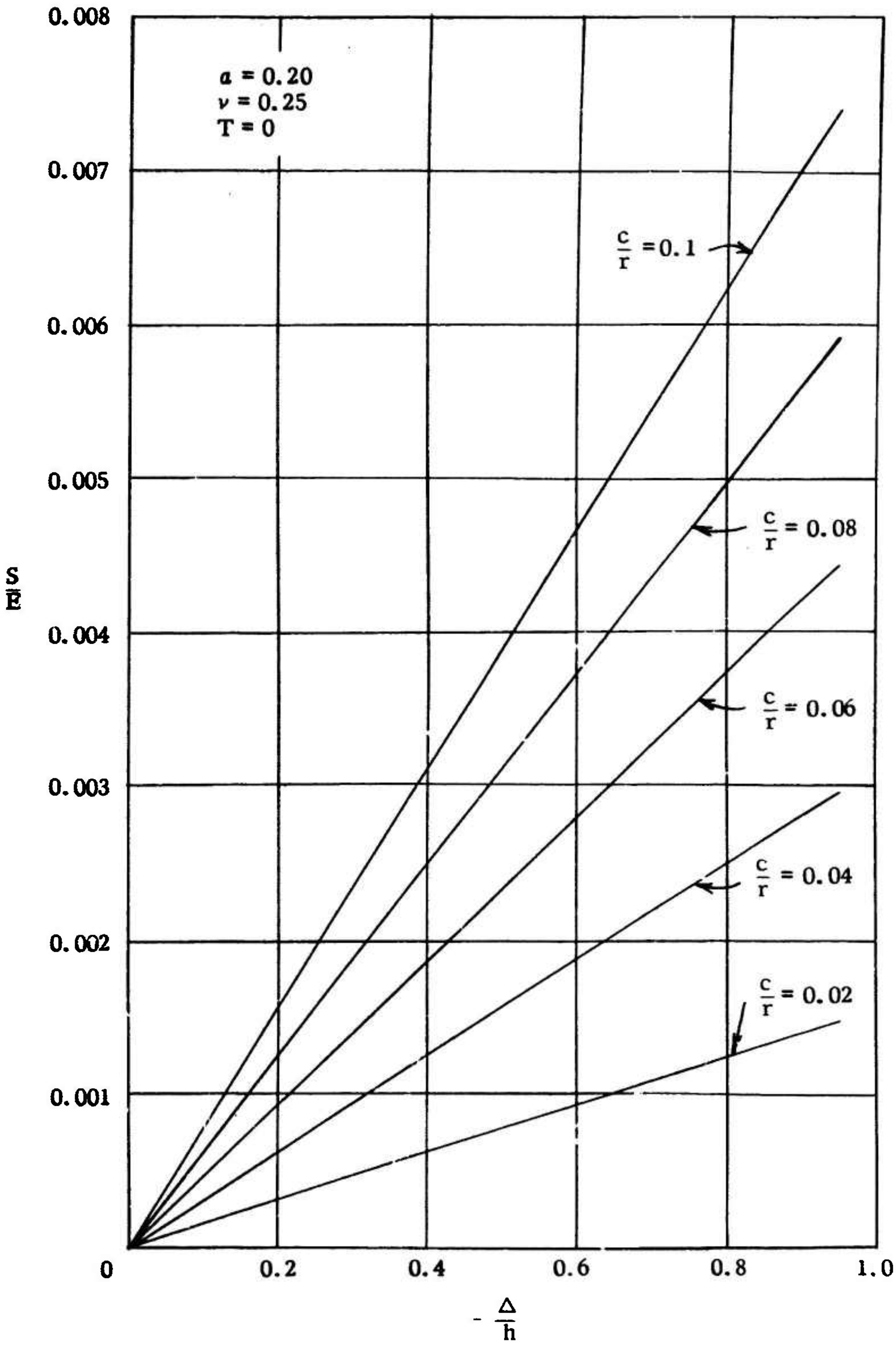


Fig. 6

Plot of $\frac{S}{E}$ vs. $\frac{\Delta}{h}$

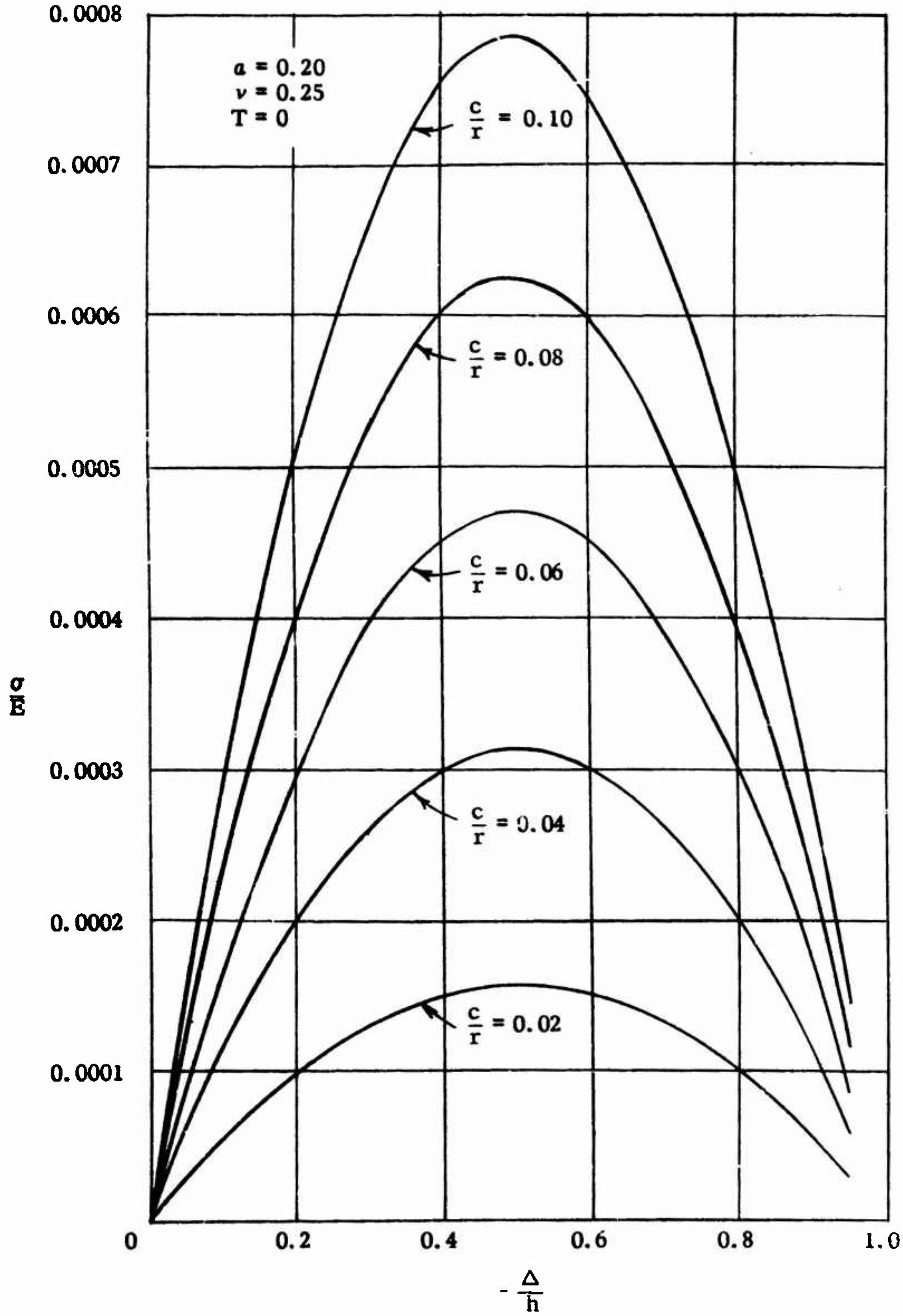


Fig. 7

Plot of $\frac{\sigma}{E}$ vs. $\frac{\Delta}{h}$

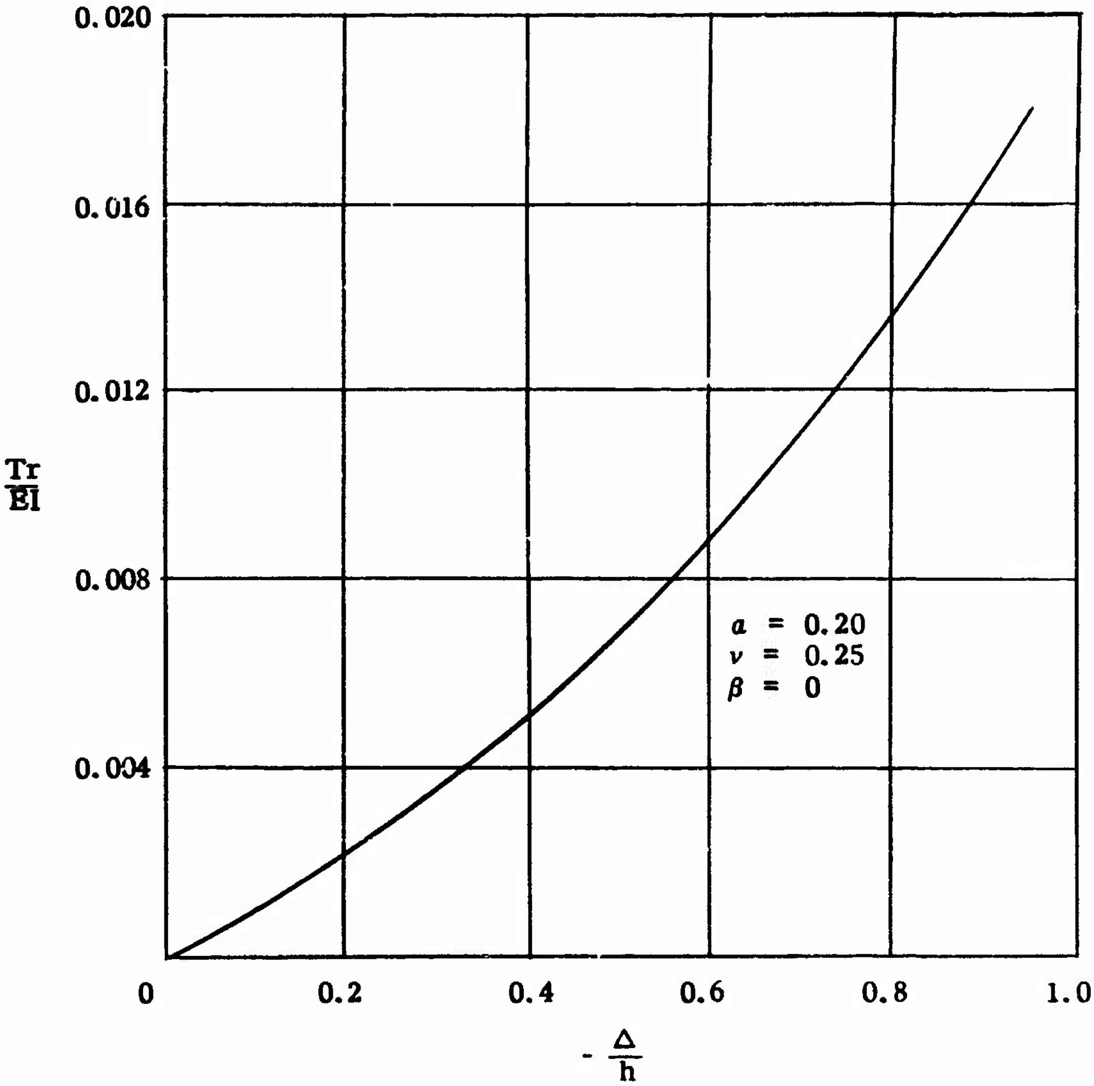


Fig. 8

Plot of $\frac{Tr}{EI}$ vs. $\frac{\Delta}{h}$

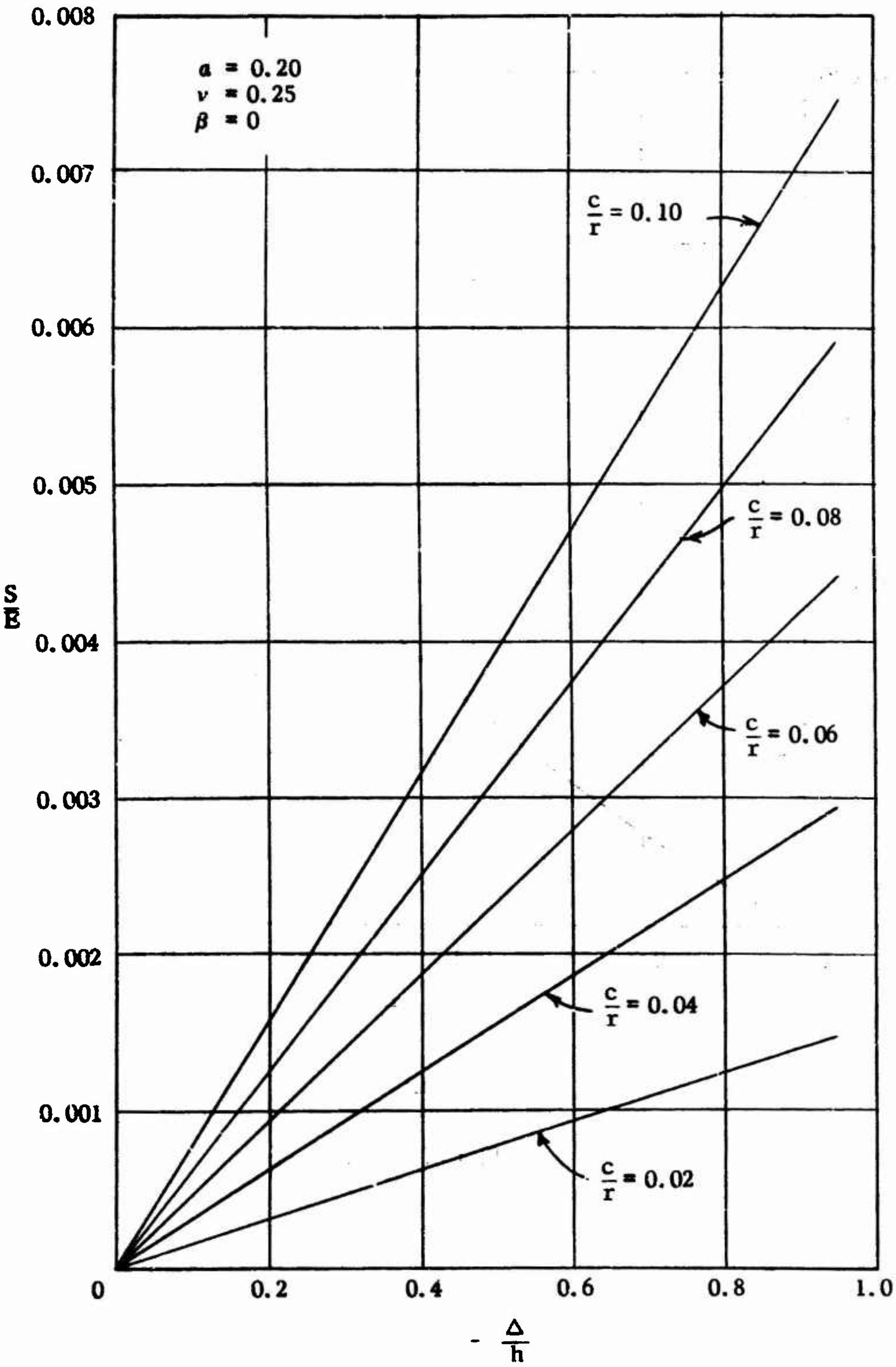


Fig. 9

Plot of $\frac{S}{E}$ vs. $\frac{\Delta}{h}$

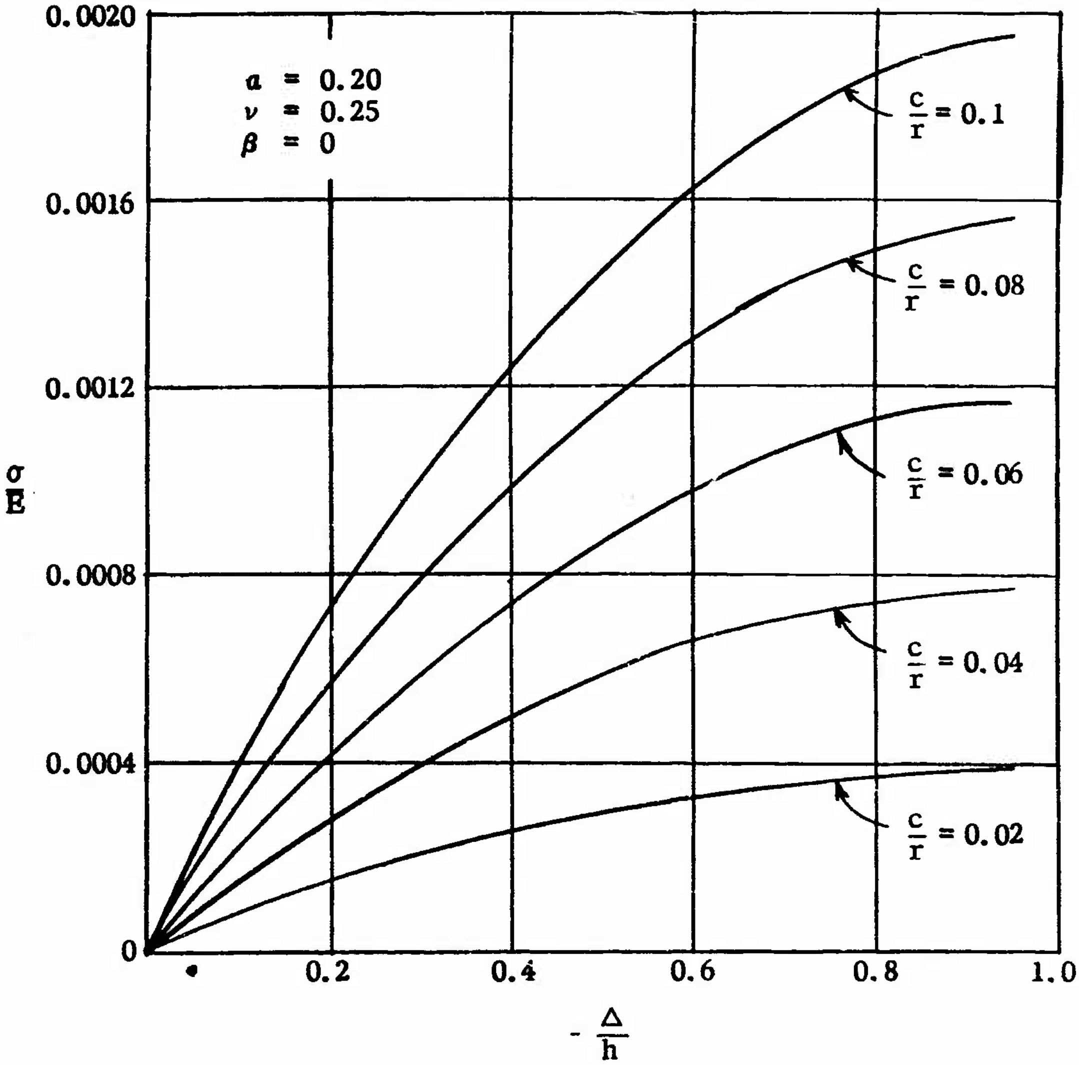


Fig. 10

Plot of $\frac{\sigma}{E}$ vs. $\frac{\Delta}{h}$

A Means of Triggering the Streak
Camera from an Event

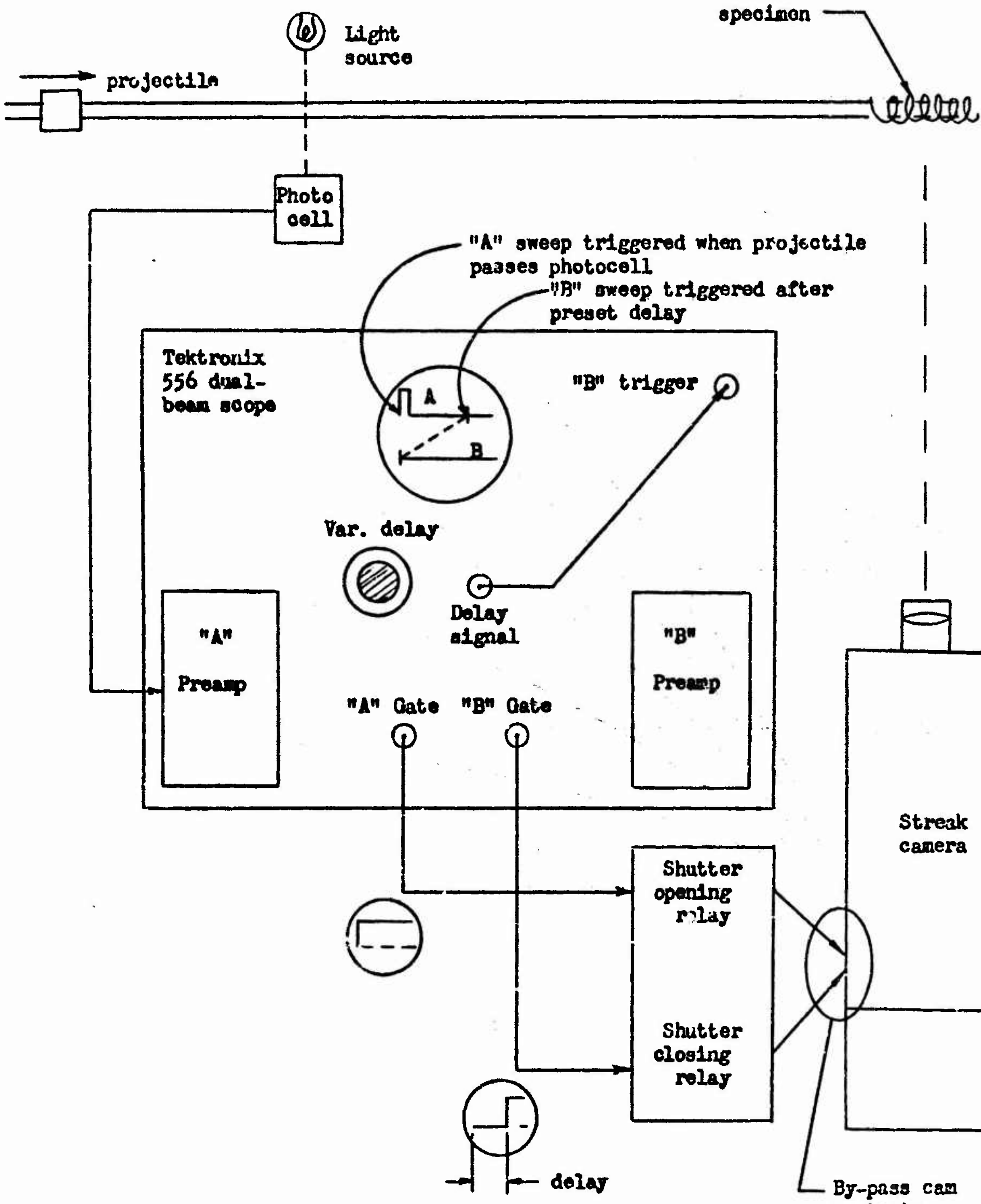


Fig. 11

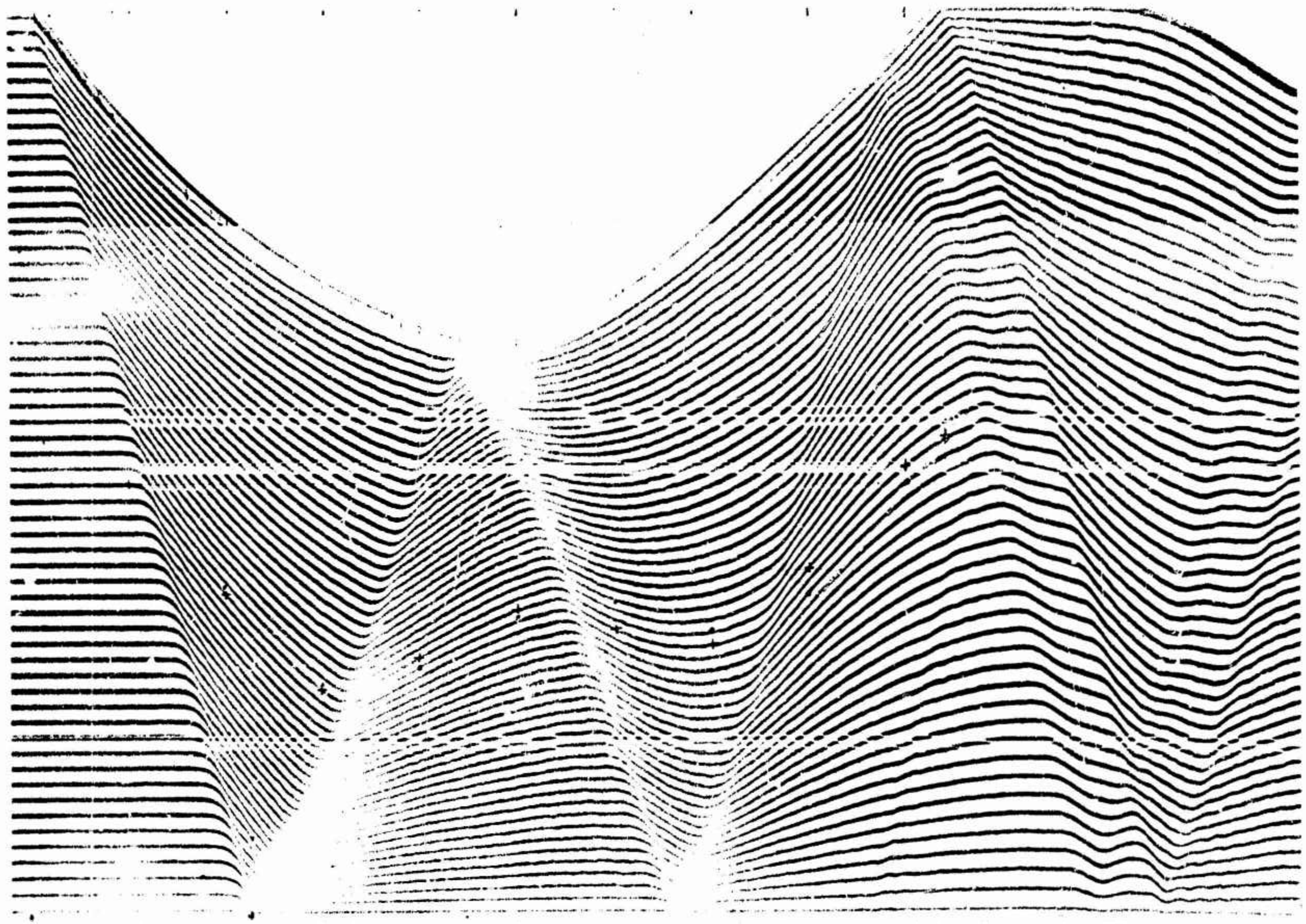
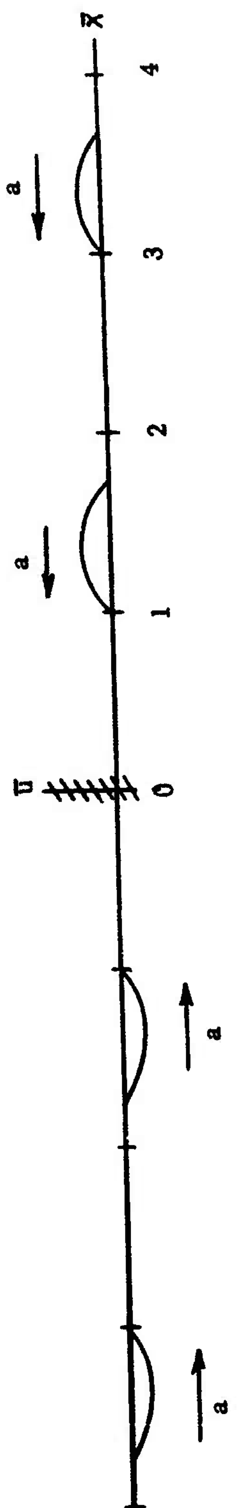


Fig. 12. Compressed Length of Spring 17.70 in, Drum Speed 2.94 revolutions per sec.
• Theory + Experiment



Solution of Wave Equation
Fig. 13

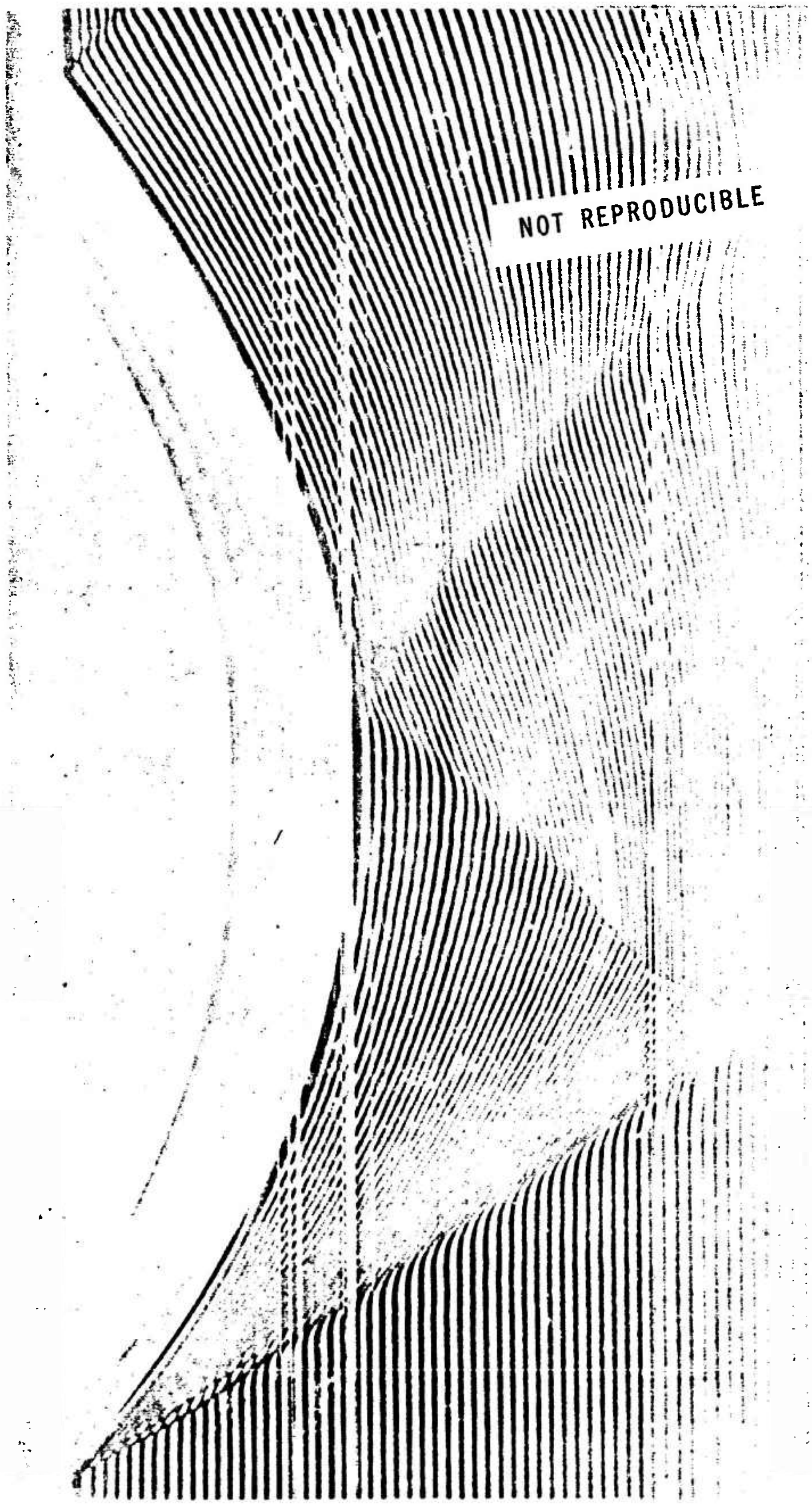


Fig. 14. Compressed Length of Spring = 12.75 in, Drum Speed 5.88 Revolution per sec,

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